

Exercise walk-through

3.1

The Chain Rule

AP Calculus AB

What makes a function composite?

A **composite function** $f(g(x))$
nests one function **inside** another.

Always ask two questions: **what is the inside**, and **what is its derivative**?



Chain rule matryoshka

The chain rule

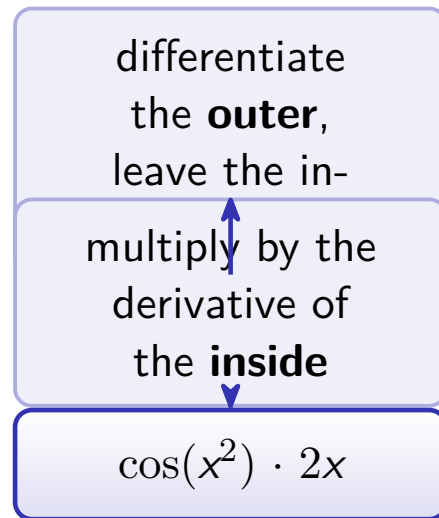
Differentiate **outer-then-inner**:

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x).$$

In Leibniz notation, with $y = f(u)$ and $u = g(x)$:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

The inner derivative $g'(x)$ is the **most-forgotten** factor on the exam.



Worked example – a power of a polynomial

Differentiate $h(x) = (2x^2 + 1)^5$

Outer is $(\cdot)^5$, inner is $2x^2 + 1$:

$$\begin{aligned} h'(x) &= 5(2x^2 + 1)^4 \cdot \frac{d}{dx}(2x^2 + 1) \\ &= 5(2x^2 + 1)^4 \cdot 4x = 20x(2x^2 + 1)^4. \end{aligned}$$

outer $(\cdot)^5 \rightarrow 5(\cdot)^4$



inner $2x^2 + 1 \rightarrow 4x$

Exam skill – the chain rule from a table

A table gives values; no formula is needed. With $h(x) = f(g(x))$:

$$h'(a) = f'(g(a)) \cdot g'(a).$$

Read $g(a)$ **first**, then look up f' at **that** value – not at a .

Find $h'(1)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	3	7	1	9

$g(1) = 2$, so

$$h'(1) = f'(2) \cdot g'(1) = 7 \times 5 = 35.$$

Exercises 2.1-2.3

2.1 State the chain rule.

[1]

2.2 Differentiate $(x^2 + 1)^3$.

[2]

2.3 State what the chain rule is for.

[1]

Exercises 2.1-2.3

2.1 State the chain rule.

[1]

2.1

- $$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

2.2 Differentiate $(x^2 + 1)^3$.

[2]

2.2

- $$3(x^2 + 1)^2 \cdot 2x = 6x(x^2 + 1)^2$$

2.3 State what the chain rule is for.

[1]

2.3

- differentiating a composite (a function of a function)

Exercises 3.1-3.2

3.1 $\frac{d}{dx}[f(g(x))]$ equals

[1]

☐

A $f'(x)g'(x)$

B $f'(g(x)) \cdot g'(x)$

C $f'(g(x))$

D $g'(f(x))$

3.2 $\frac{d}{dx}[\sin(x^2)]$ equals

[1]

☐

A $\cos(x^2)$

B $2x \cos(x^2)$

C $2x \sin(x^2)$

D $\cos(2x)$

Exercises 3.1-3.2

3.1 $\frac{d}{dx}[f(g(x))]$ equals

[1]

☐

- A $f'(x)g'(x)$
- B $f'(g(x)) \cdot g'(x)$
- C $f'(g(x))$
- D $g'(f(x))$

3.1 B

- $f'(g(x)) \cdot g'(x)$.

3.2 $\frac{d}{dx}[\sin(x^2)]$ equals

[1]

☐

- A $\cos(x^2)$
- B $2x \cos(x^2)$
- C $2x \sin(x^2)$
- D $\cos(2x)$

3.2 B

- $2x \cos(x^2)$.

Exercise 3.3 (a)

3.3 $y = (3x + 1)^4$.

(a) Identify the outer and inner functions.

[1]

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(a) Identify the outer and inner functions.

[1]

(a)

- outer u^4 , inner $3x + 1$

Exercise 3.3 (b)

3.3 $y = (3x + 1)^4$.

(b) Apply the chain rule.

[1]

Exercise 3.3 (b)

3.3 $y = (3x + 1)^4$.

(b) Apply the chain rule.

[1]

(b)

- $4(3x + 1)^3 \cdot 3$

Exercise 3.3 (c)

3.3 $y = (3x + 1)^4$.

(c) State y' .

[1]

Exercise 3.3 (c)

3.3 $y = (3x + 1)^4$.

(c) State y' .

[1]

(c)

- $y' = 12(3x + 1)^3$