

Exercise walk-through

2.3

Estimating Derivatives of a Function at a Point

AP Calculus AB

From secant to tangent

Let the point a vary and the derivative becomes a **new function**:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

This is the **definition of the derivative** (sometimes called differentiating "by first...

Its value at each x is the instantaneous rate of change there.

The notations, and the tangent line

the notations $\frac{dy}{dx}$, $f'(x)$, y' – all naming the same object

which to use Leibniz $\frac{dy}{dx}$ when the variables matter; Lagrange $f'(x)$ when the function does

the second derivative $\frac{d^2y}{dx^2}$ or $f''(x)$

the tangent line give it as $y - f(a) = f'(a)(x - a)$

why that form it needs no rearrangement and no risk of an arithmetic slip in the intercept

the check substitute $x = a$: the equation must give $y = f(a)$

Writing the tangent in point-slope form is a marked convention, not a preference. Converting it to $y = mx + c$ adds a step and an opportunity to lose a mark.

Differentiability is the stronger condition

the relationship

differentiable $\Rightarrow$ continuous

why

a derivative is a limit that cannot exist where the function jumps

the converse is false

a continuous function can fail to be differentiable

a corner

$|x|$ at $x = 0$ – the left and right slopes differ

a cusp

$x^{2/3}$ at $x = 0$ – the slopes go to $+\infty$ and $-\infty$

a vertical tangent

$x^{1/3}$ at $x = 0$ – the slope is infinite, so no finite derivative

On a justification question, say **which direction** of the implication you are using. “It is continuous, therefore differentiable” is the error the question is set to catch.

Exercises 2.1-2.3

2.1 State a formula to estimate $f'(a)$ from a table. [1]

2.2 If $f(2) = 5$ and $f(2.1) = 5.4$, estimate $f'(2)$. [2]

2.3 State why a smaller h generally gives a better estimate. [1]

Exercises 2.1·2.3

2.1 State a formula to estimate $f'(a)$ from a table.

[1]

2.1

- $f'(a) \approx \frac{f(a+h) - f(a)}{h}$ for small h

2.2 If $f(2) = 5$ and $f(2.1) = 5.4$, estimate $f'(2)$.

[2]

2.2

- $\frac{5.4 - 5}{2.1 - 2} = \frac{0.4}{0.1} = 4$

2.3 State why a smaller h generally gives a better estimate.

[1]

2.3

- a smaller interval is closer to the tangent slope (the limit)

Exercise 3.1

3.1 A derivative at a point can be estimated from a table using a

[1]

☐

A single value

B difference quotient with small h

C maximum

D mean

Exercise 3.1

3.1 A derivative at a point can be estimated from a table using a

[1]

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A single value

B difference quotient with small h

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3.1 B

- difference quotient with small h .

Exercise 3.2

3.2 The symmetric difference quotient is

[1]

☐

A $\frac{f(a+h) - f(a)}{h}$

B $\frac{f(a+h) - f(a-h)}{2h}$

C $\frac{f(a)}{h}$

D $f(a+h) \cdot f(a-h)$

Exercise 3.2

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[1]



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B $\frac{f(a+h) - f(a-h)}{2h}$

C $\frac{f(a)}{h}$

D $f(a+h) \cdot f(a-h)$

3.2 B

• $\frac{f(a+h) - f(a-h)}{2h}.$

Exercise 3.3 (a)

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(a) Find Δf .

[1]

Exercise 3.3 (a)

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(a) Find Δf .

[1]

(a)

- 0.6

Exercise 3.3 (b)

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(b) Find Δx .

[1]

Exercise 3.3 (b)

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(b) Find Δx .

[1]

(b)

- 0.2

Exercise 3.3 (c)

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(c) Estimate $f'(3)$.

[1]

Exercise 3.3 (c)

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(c) Estimate $f'(3)$.

[1]

(c)

- $\frac{0.6}{0.2} = 3$