

Exercise walk-through

2.2

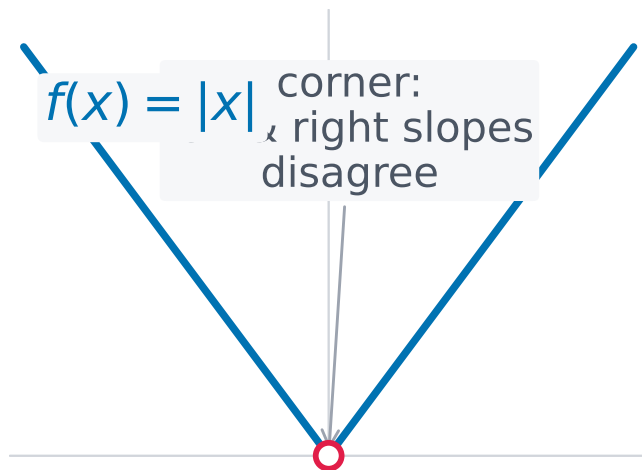
Defining the Derivative of a Function and Using Derivative Notation

AP Calculus AB

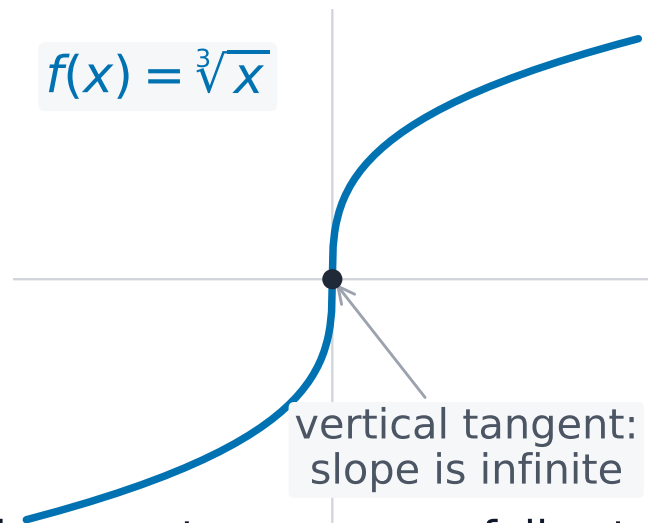
Differentiable $\Rightarrow$ continuous (not the reverse)

If f is **differentiable** at a point, then f is **continuous** there. A continuous function can still **fail** to be differentiable:

continuous, not differentiable continuous, not differentiable



$$f(x) = \sqrt[3]{x}$$



f differentiable at $a \Rightarrow f$ continuous at a ; converse fails at corners and vertical tangents

Use the contrapositive: if f is **not continuous** at a , it is **not differentiable** at a .

The notations, and the tangent line

the notations $\frac{dy}{dx}$, $f'(x)$, y' – all naming the same object

which to use Leibniz $\frac{dy}{dx}$ when the variables matter; Lagrange $f'(x)$ when the function does

the second derivative $\frac{d^2y}{dx^2}$ or $f''(x)$

the tangent line give it as $y - f(a) = f'(a)(x - a)$

why that form it needs no rearrangement and no risk of an arithmetic slip in the intercept

the check substitute $x = a$: the equation must give $y = f(a)$

Writing the tangent in point-slope form is a marked convention, not a preference. Converting it to $y = mx + c$ adds a step and an opportunity to lose a mark.

Differentiability is the stronger condition

the relationship

differentiable $\Rightarrow$ continuous

why

a derivative is a limit that cannot exist where the function jumps

the converse is false

a continuous function can fail to be differentiable

a corner

$|x|$ at $x = 0$ – the left and right slopes differ

a cusp

$x^{2/3}$ at $x = 0$ – the slopes go to $+\infty$ and $-\infty$

a vertical tangent

$x^{1/3}$ at $x = 0$ – the slope is infinite, so no finite derivative

On a justification question, say **which direction** of the implication you are using. “It is continuous, therefore differentiable” is the error the question is set to catch.

Exercises 2.1·2.3

2.1 Write the limit definition of the derivative.

[1]

2.2 State two notations for the derivative.

[1]

2.3 State what $f'(x)$ measures geometrically.

[2]

Exercises 2.1-2.3

2.1 Write the limit definition of the derivative.

[1]

2.1

- $$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2.2 State two notations for the derivative.

[1]

2.2

- any two of $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$

2.3 State what $f'(x)$ measures geometrically.

[2]

2.3

- the slope of the tangent line to the graph of f at x

Exercise 3.1

3.1 The derivative $f'(x)$ is defined as

[1]

☐

A $f(x)/x$

B $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

C $f(x) \cdot x$

D $\frac{f(b) - f(a)}{b - a}$ only

Exercise 3.1

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D $\frac{f(b) - f(a)}{b - a}$ only

3.1 B

• $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$

Exercise 3.2

3.2 $\frac{dy}{dx}$ is another notation for the

[1]

☐

A integral

B derivative

C limit only

D area

Exercise 3.2

3.2 $\frac{dy}{dx}$ is another notation for the

[1]

☐

A integral

B derivative

C limit only

D area

3.2 B

- derivative.

Exercise 3.3 (a)

3.3 Answer each.

(a) $f'(a)$ is the slope of the _____ line.

[1]

Exercise 3.3 (a)

3.3 Answer each.

(a) $f'(a)$ is the slope of the _____ line.

[1]

(a)

- tangent

Exercise 3.3 (b)

3.3 Answer each.

(b) Name the quantity $f'(x)$.

[1]

Exercise 3.3 (b)

3.3 Answer each.

(b) Name the quantity $f'(x)$.

[1]

(b)

- the derivative (instantaneous rate of change)

Exercise 3.3 (c)

3.3 Answer each.

(c) State what $\frac{dy}{dx}$ reads as.

[1]

Exercise 3.3 (c)

3.3 Answer each.

(c) State what $\frac{dy}{dx}$ reads as.

[1]

(c)

- the rate of change of y with respect to x