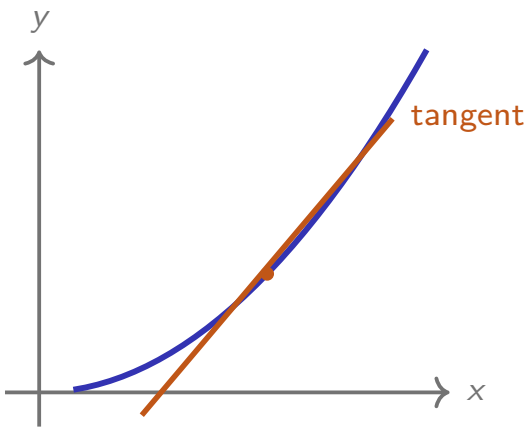
Exercise walk-through

2.1 Defining Average and Instantaneous Rates of Change at a Point

AP Calculus AB

What we will cover

1. Average vs instantaneous rate
2. The derivative as a limit
3. When a derivative exists
4. Basic derivative rules
5. Product & quotient rules
6. Trig, exp & log derivatives



From secant to tangent

Let the point a vary and the derivative becomes a **new function**:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

This is the **definition of the derivative** (sometimes called differentiating "by first...

> Its value at each x is the instantaneous rate of change there.

The notations, and the tangent line

the notations	$\frac{dy}{dx}$, $f'(x)$, y' – all naming the same object
which to use	Leibniz $\frac{dy}{dx}$ when the variables matter; Lagrange $f'(x)$ when the function does
the second derivative	$\frac{d^2y}{dx^2}$ or $f''(x)$
the tangent line	give it as $y - f(a) = f'(a)(x - a)$
why that form	it needs no rearrangement and no risk of an arithmetic slip in the intercept
the check	substitute $x = a$: the equation must give $y = f(a)$

> Writing the tangent in point–slope form is a marked convention, not a preference. Converting it to $y = mx + c$ adds a step and an opportunity to lose a mark.

Exercises 2.1·2.3

2.1 State the difference quotient for the average rate over $[a, a+h]$. [1]

2.2 State what happens to it as $h \to 0$. [1]

2.3 For $f(x) = x^2$, compute $\dfrac{f(2+h) - f(2)}{h}$ and simplify. [2]

Exercises 2.1·2.3

2.1 State the difference quotient for the average rate over $[a, a+h]$. [1]

2.1

- $\dfrac{f(a+h)-f(a)}{h}$

2.2 State what happens to it as $h \to 0$. [1]

2.2

- it approaches the instantaneous rate of change (the derivative)

2.3 For $f(x) = x^2$, compute $\dfrac{f(2+h)-f(2)}{h}$ and simplify. [2]

2.3

- $\dfrac{(2+h)^2-4}{h} = \dfrac{4h+h^2}{h} = 4+h$

Exercises 3.1·3.2

3.1 The average rate of change over $[a, a+h]$ is [1]

- **A** $f(a)h$
- **B** $\dfrac{f(a+h)-f(a)}{h}$
- **C** $f'(a)$
- **D** h

3.2 The instantaneous rate is the average rate as [1]

- **A** $h \to \infty$
- **B** $h \to 0$
- **C** $h = 1$
- **D** $a \to 0$

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3.1 B

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3.2 B

- $h \to 0$.

Exercise 3.3 (a)

3.3 $f(x) = x^2$ at $a = 3$.

(a) Write $f(3 + h)$. [1]

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(a) Write $f(3 + h)$. [1]

(a)

- $(3 + h)^2$

Exercise 3.3 (b)

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(b) Form the difference quotient. [1]

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(b)

- $\dfrac{(3+h)^2 - 9}{h} = 6 + h$

Exercise 3.3 (c)

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(c) State its limit as $h \to 0$. [1]

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(c)

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