

Exercise walk-through

1.4

Estimating Limit Values from Tables

AP Calculus AB

Defining a limit

$\lim_{x \rightarrow c} f(x) = R$ means $f(x)$ can be made
arbitrarily close to R by taking x
sufficiently close to c – but not equal
to c .

A limit describes the **behaviour near** c ,
not the value at c .

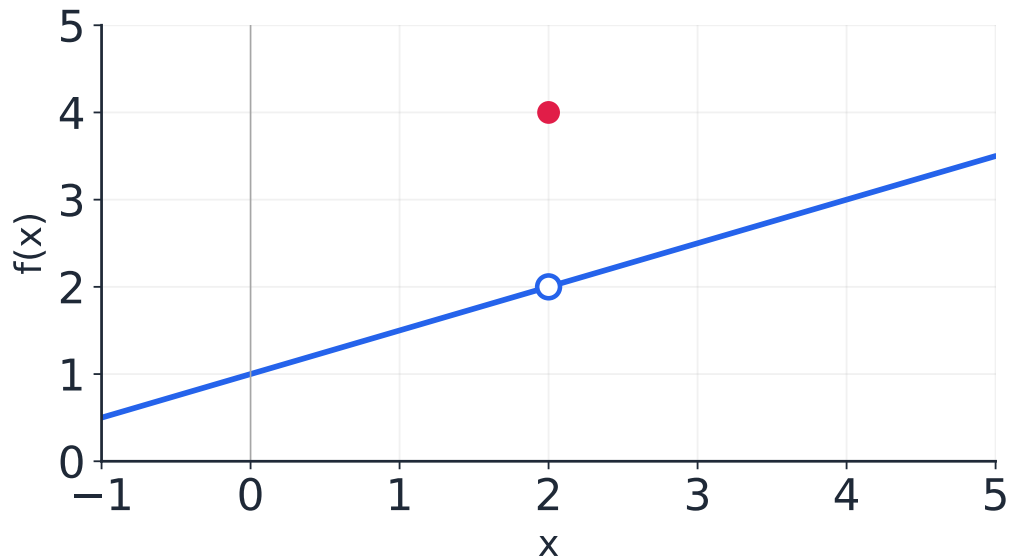
graphically

numerically

analytically

Moving between the three is a core skill.

Read the curve and the filled point separately



Your turn, at $x = 2$

Find the left-hand limit, right-hand limit, two-sided limit and $f(2)$. Is the function continuous? Explain.

Trace nearby inputs first. Read the filled point only for the function value.

The limit is two, while the value is four

$$\lim_{x \rightarrow 2^-} f(x) = 2, \quad \lim_{x \rightarrow 2^+} f(x) = 2.$$

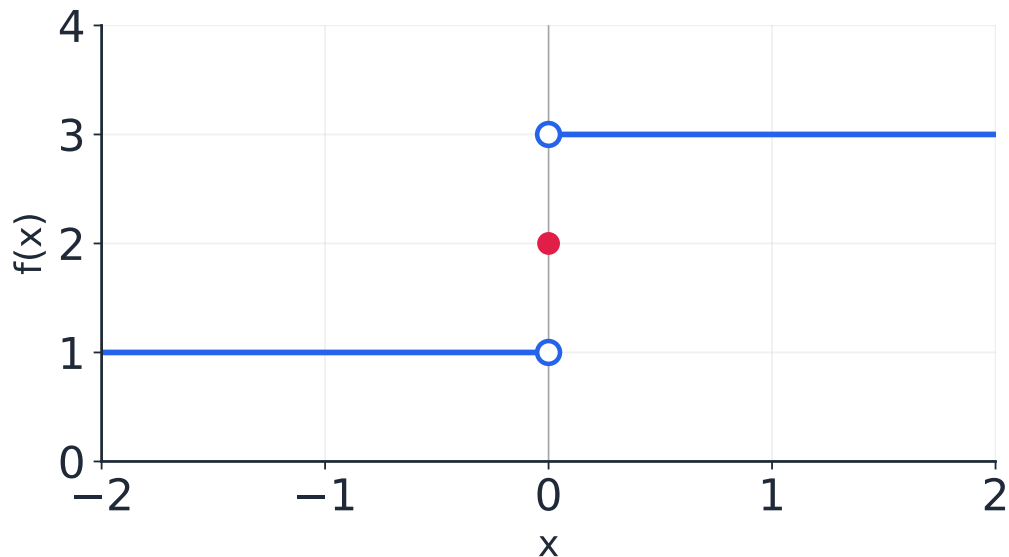
$$\lim_{x \rightarrow 2} f(x) = 2, \quad f(2) = 4.$$

Why the distinction matters

Both branches approach height 2, so the two-sided limit exists. The filled point gives 4. The function is discontinuous because the limit and value differ.

Redefining $f(2) = 2$ repairs this removable discontinuity; nearby values stay unchanged.

A filled point does not join unequal limits



Your turn, at $x = 0$

Find both one-sided limits and $f(0)$. Does a two-sided limit exist? Can changing $f(0)$ make the function continuous?

The existence of a filled point does not establish a two-sided limit.

Changing one value cannot remove a jump

$$\lim_{x \rightarrow 0^-} f(x) = 1, \quad \lim_{x \rightarrow 0^+} f(x) = 3, \quad f(0) = 2.$$

Reason from the approaching heights

Since $1 \neq 3$, the two-sided limit does not exist. This is a jump discontinuity. Any new value at 0 leaves the two nearby branches unchanged, so no single point value repairs it.

Use **left-hand limit** and **right-hand limit** as evidence. An **open circle** marks an excluded point; a **closed circle** gives the actual value.

Exercises 2.1·2.3

2.1 State how a table is used to estimate a limit. [1]

2.2 From $f(1.9) = 3.8$, $f(1.99) = 3.98$, $f(2.01) = 4.02$, estimate $\lim_{x \rightarrow 2} f(x)$. [2]

2.3 State why you check both sides of a . [1]

Exercises 2.1-2.3

2.1 State how a table is used to estimate a limit. [1]

2.1

- tabulate $f(x)$ for x near a and see what value the outputs approach
-

2.2 From $f(1.9) = 3.8$, $f(1.99) = 3.98$, $f(2.01) = 4.02$, estimate $\lim_{x \rightarrow 2} f(x)$. [2]

2.2

- the values approach 4, so $\lim_{x \rightarrow 2} f(x) \approx 4$
-

2.3 State why you check both sides of a . [1]

2.3

- to confirm the left- and right-hand limits agree

Exercises 3.1-3.2

3.1 To estimate a limit from a table, pick x -values

[1]

☐

A far from a

B approaching a from both sides

C only greater than a

D equal to a

☐

3.2 If $f(x)$ approaches 7 from both sides, the limit is

[1]

A 0

B 7

C undefined

D 14

Exercises 3.1-3.2

3.1 To estimate a limit from a table, pick x -values

[1]

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A far from a

B approaching a from both sides

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D equal to a

3.1 B

- approaching a from both sides.

3.2 If $f(x)$ approaches 7 from both sides, the limit is

[1]

☐

A 0

B 7

C undefined

D 14

3.2 B

- 7.

Exercise 3.3 (a)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(a) State the left-hand trend.

[1]

Exercise 3.3 (a)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(a) State the left-hand trend.

[1]

(a)

- approaching 6

Exercise 3.3 (b)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(b) State the right-hand trend.

[1]

Exercise 3.3 (b)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(b) State the right-hand trend.

[1]

(b)

- approaching 6

Exercise 3.3 (c)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(c) State $\lim_{x \rightarrow 3} f(x)$.

[1]

Exercise 3.3 (c)

3.3 Table: $f(2.9) = 5.9$, $f(2.99) = 5.99$, $f(3.01) = 6.01$.

(c) State $\lim_{x \rightarrow 3} f(x)$.

[1]

(c)

- 6

Exercise 3.4

3.4 At $x = 3.9, 3.99, 4.01, 4.1$, a function has values 7.9, 7.99, 8.01, 8.1. (a) Estimate the limit at 4 using both sides. [2]

(b) Does the table prove continuity at 4? Give two reasons for your answer. [2]

Exercise 3.4

3.4 At $x = 3.9, 3.99, 4.01, 4.1$, a function has values $7.9, 7.99, 8.01, 8.1$. (a) Estimate the limit at 4 using both sides. [2]

(b) Does the table prove continuity at 4? Give two reasons for your answer. [2]

3.4

(a) Both trends approach 8, so the estimated limit is 8.

(b) No: the value at 4 is not given, and finitely many sampled values do not prove the limiting behaviour at every sufficiently close input.