

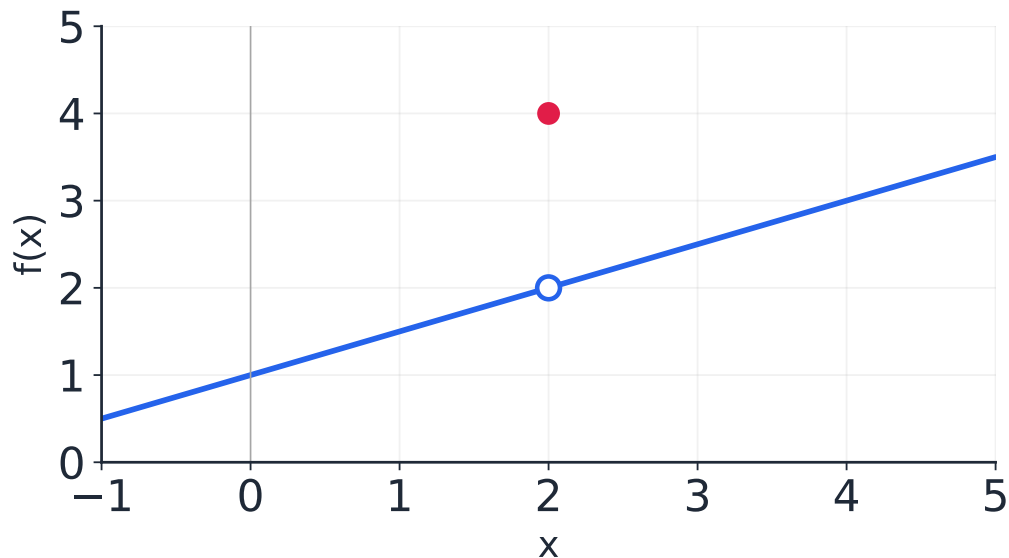
Exercise walk-through

1.3

Estimating Limit Values from Graphs

AP Calculus AB

Read the curve and the filled point separately



Your turn, at $x = 2$

Find the left-hand limit, right-hand limit, two-sided limit and $f(2)$. Is the function continuous? Explain.

Trace nearby inputs first. Read the filled point only for the function value.

The limit is two, while the value is four

$$\lim_{x \rightarrow 2^-} f(x) = 2, \quad \lim_{x \rightarrow 2^+} f(x) = 2.$$

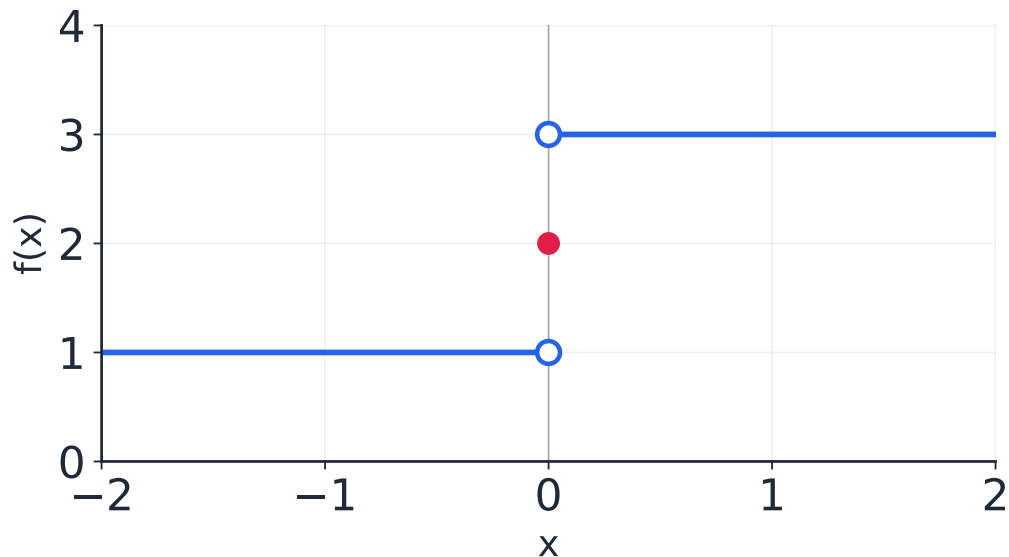
$$\lim_{x \rightarrow 2} f(x) = 2, \quad f(2) = 4.$$

Why the distinction matters

Both branches approach height 2, so the two-sided limit exists. The filled point gives 4. The function is discontinuous because the limit and value differ.

Redefining $f(2) = 2$ repairs this removable discontinuity; nearby values stay unchanged.

A filled point does not join unequal limits



Your turn, at $x = 0$

Find both one-sided limits and $f(0)$. Does a two-sided limit exist? Can changing $f(0)$ make the function continuous?

The existence of a filled point does not establish a two-sided limit.

Changing one value cannot remove a jump

$$\lim_{x \rightarrow 0^-} f(x) = 1, \quad \lim_{x \rightarrow 0^+} f(x) = 3, \quad f(0) = 2.$$

Reason from the approaching heights

Since $1 \neq 3$, the two-sided limit does not exist. This is a jump discontinuity. Any new value at 0 leaves the two nearby branches unchanged, so no single point value repairs it.

Use **left-hand limit** and **right-hand limit** as evidence. An **open circle** marks an excluded point; a **closed circle** gives the actual value.

Exercises 2.1-2.3

2.1 State when a two-sided limit exists.

[1]

2.2 State the notation for a left-hand limit.

[1]

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit.

[2]

Exercises 2.1-2.3

2.1 State when a two-sided limit exists. [1]

2.1

- when the left-hand and right-hand limits are equal
-

2.2 State the notation for a left-hand limit. [1]

2.2

- $\lim_{x \rightarrow a^-} f(x)$
-

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit. [2]

2.3

- the two-sided limit is 4

Exercises 3.1·3.2

3.1 A two-sided limit exists if and only if

[1]

☐

A $f(a)$ is defined

B the left and right limits are equal

C the graph is a straight line

D the limit is 0

3.2 A hole in the graph at $x = a$ means

[1]

☐

A the limit cannot exist

B the limit may still exist

C f is linear

D $a = 0$

Exercises 3.1-3.2

3.1 A two-sided limit exists if and only if

[1]

☐

A $f(a)$ is defined

B the left and right limits are equal

C the graph is a straight line

D the limit is 0

3.1 B

- the left and right limits are equal.

3.2 A hole in the graph at $x = a$ means

[1]

☐

A the limit cannot exist

B the limit may still exist

C f is linear

D $a = 0$

3.2 B

- the limit may still exist.

Exercise 3.3 (a)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit.

[1]

Exercise 3.3 (a)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit.

[1]

(a)

- 5

Exercise 3.3 (b)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(b) State the right-hand limit.

[1]

Exercise 3.3 (b)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(b) State the right-hand limit.

[1]

(b)

- 3

Exercise 3.3 (c)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(c) State whether the two-sided limit exists.

[1]

Exercise 3.3 (c)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(c) State whether the two-sided limit exists.

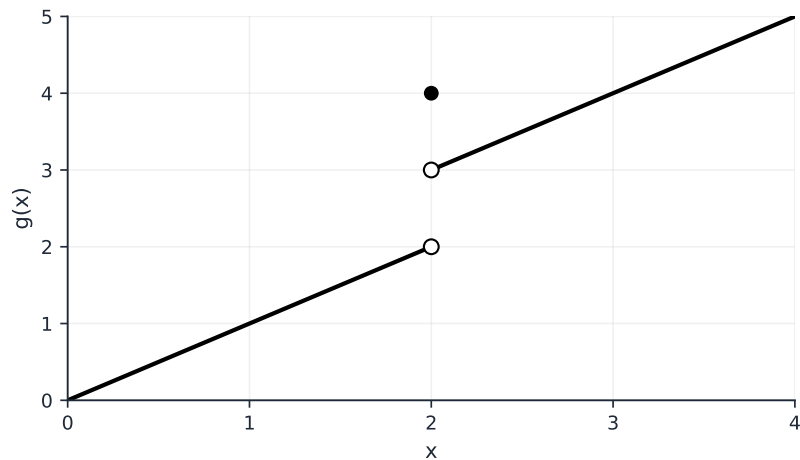
[1]

(c)

- no, the one-sided limits differ

Setup for exercise 3.4

3.4 The graph defines g near $x = 2$.



Exercise 3.4 (a)

(a) State its left- and right-hand limits at 2.

[2]

Exercise 3.4 (a)

(a) State its left- and right-hand limits at 2.

[2]

(a) The left-hand limit is 2 and the right-hand limit is 3.

Exercise 3.4 (b)

(b) Does the two-sided limit exist? Explain using the graph.

[2]

Exercise 3.4 (b)

- (b) Does the two-sided limit exist? Explain using the graph. [2]
- (b) The two-sided limit does not exist, because the one-sided approaching heights differ. The filled point at 4 does not resolve that mismatch.