Exercise walk-through

1.15 Connecting Limits at Infinity and Horizontal Asymptotes

AP Calculus AB

Infinite limits and vertical asymptotes

Where a non-zero number is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote** .

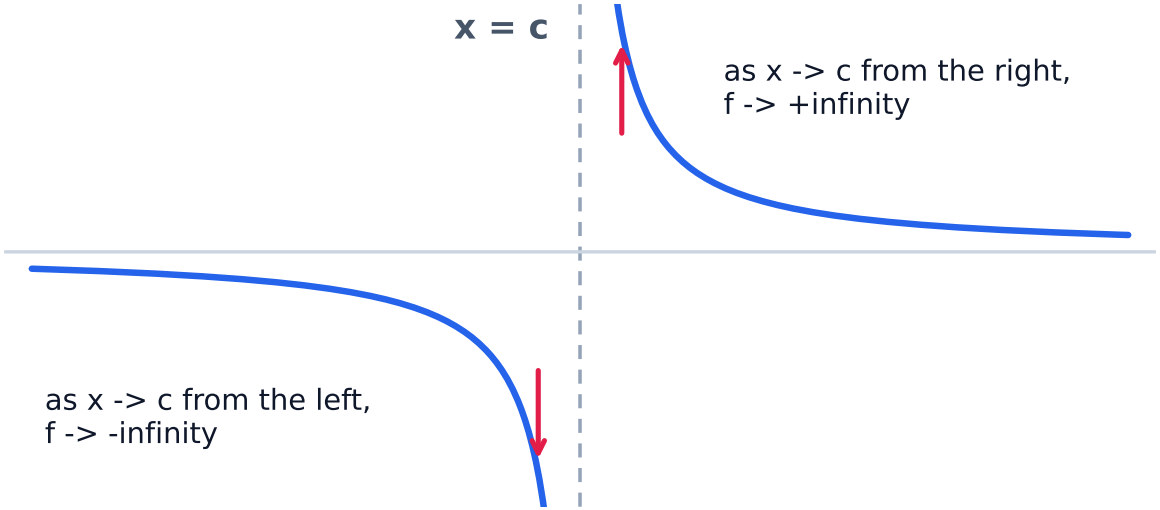
> Always check each side: the two sides can shoot opposite ways.

End behaviour, from the leading terms

Relative magnitudes	At infinity only the **relative magnitudes** matter: exponentials beat any polynomial, and a polynomial beats any logarithm.
Equal degrees	When numerator and denominator have the same degree, the limit is the **ratio of the leading coefficients** – and that ratio is the horizontal asymptote.
Unequal degrees	Numerator smaller: the limit is 0. Numerator larger: it grows without bound, and polynomial division gives the slant asymptote.
Naming it	That long-run **behavior** – the **end behavior** – is what a limit at infinity reports.

> Divide every term by the highest power in the denominator. Every rational end-behaviour limit falls out of that one step.

Infinite limits and vertical asymptotes, in detail



vertical asymptote $x = c$ when $\lim_{x \to c^+} f(x) = \pm\infty$ or $\lim_{x \to c^-} f(x) = \pm\infty$

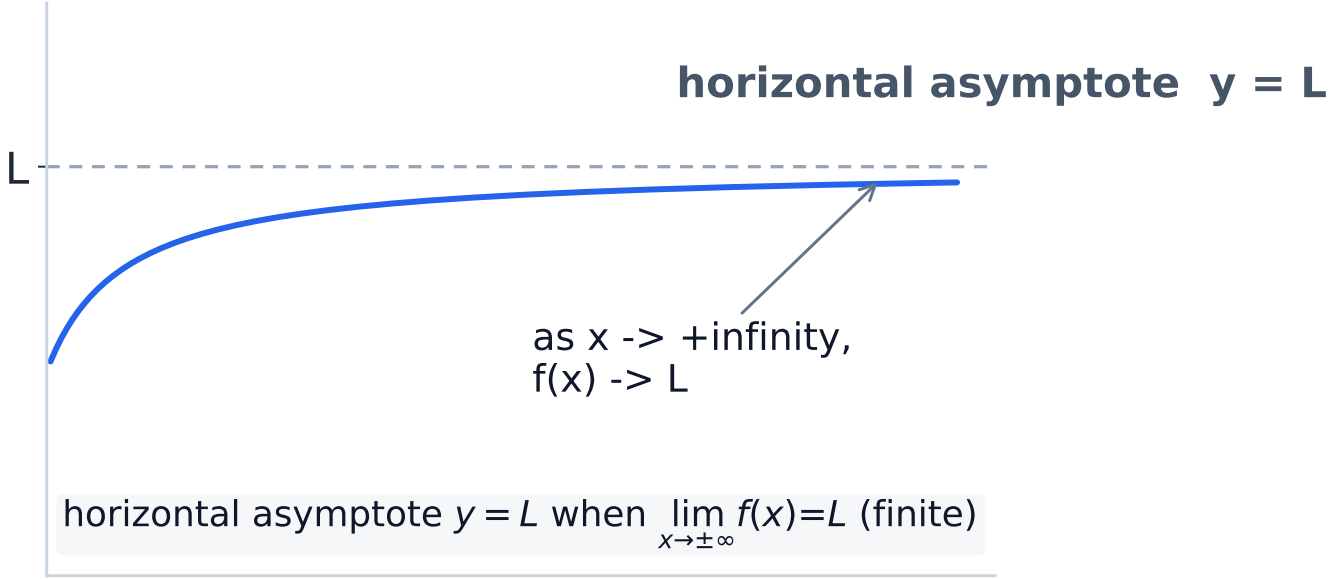
Where a non-zero number is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote** .

Limits at infinity and horizontal asymptotes

Let the input grow: **end behaviour** as $x \to \pm\infty$. If outputs settle toward L, then $y = L$ is a **horizontal asymptote** .

> Far out, an exponential beats any polynomial, and a polynomial beats any logarithm.

Limits at infinity and horizontal asymptotes, in detail



Let the input grow and watch the **end behaviour** : if the outputs settle towards L, then $y = L$ is a **horizontal asymptote** .

Exercises 2.1·2.3

2.1 State what $\lim_{x\to\infty} f(x) = L$ indicates. [1]

2.2 For $f(x) = \dfrac{1}{x}$, state $\lim_{x\to\infty} f(x)$. [1]

2.3 For $f(x) = \dfrac{2x}{x+1}$, state the horizontal asymptote. [2]

Exercises 2.1·2.3

2.1 State what $\lim_{x\to\infty} f(x) = L$ indicates. [1]

2.1

- the graph has a horizontal asymptote $y = L$

2.2 For $f(x) = \dfrac{1}{x}$, state $\lim_{x\to\infty} f(x)$. [1]

2.2

- 0

2.3 For $f(x) = \dfrac{2x}{x+1}$, state the horizontal asymptote. [2]

2.3

- $y = 2$ (equal degrees, ratio $2/1$)

Exercises 3.1·3.2

3.1 $\lim_{x\to\infty} f(x) = L$ means $y = L$ is a [1]

- **A** vertical asymptote
- **B** horizontal asymptote
- **C** hole
- **D** root

3.2 $\lim_{x\to\infty} \frac{1}{x}$ equals [1]

- **A** 1
- **B** 0
- **C** ∞
- **D** -1

Exercises 3.1·3.2

3.1 $\lim_{x\to\infty} f(x) = L$ means $y = L$ is a [1]

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- **D** root

3.1 B

- horizontal asymptote.

3.2 $\lim_{x\to\infty} \frac{1}{x}$ equals [1]

- ☐ **A** 1
- **B** 0
- **C** ∞
- **D** -1

3.2 B

- 0.

Exercise 3.3 (a)

3.3 $f(x) = \dfrac{3x^2}{x^2+1}$.

(a) Compare the degrees. [1]

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(a)

- equal (both degree 2)

Exercise 3.3 (b)

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(b) Take the ratio of the leading coefficients. [1]

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(b)

- $3/1 = 3$

Exercise 3.3 (c)

3.3 $f(x) = \dfrac{3x^2}{x^2+1}$.

(c) State the horizontal asymptote. [1]

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(c) State the horizontal asymptote. [1]

(c)

- $y = 3$

Exercise 3.4

3.4 (a) Find $\lim_{x\to\infty}(2x^2 - x)/(5x^2 + 1)$, showing your algebraic method. [2]

(b) State the horizontal asymptote and explain whether it determines the value at $x = 0$. [2]

Exercise 3.4

3.4 (a) Find $\lim_{x\to\infty}(2x^2 - x)/(5x^2 + 1)$, showing your algebraic method. [2]

(b) State the horizontal asymptote and explain whether it determines the value at $x = 0$. [2]

3.4

(a) Divide by x^2: the limit is $(2 - 1/x)/(5 + 1/x^2) \to 2/5$.

(b) $y = 2/5$; it describes end behaviour, whereas direct substitution gives $g(0) = 0$.