

Exercise walk-through

1.13

Removing Discontinuities

AP Calculus AB

What calculus is for, and the words limits are described in

calculus

the mathematics of **change** and of **accumulation**

the two questions

how fast something changes **at an instant**, and how much accumulates **over an interval**

a limit

the value $f(x)$ approaches as x approaches a – and it **ignores the point itself**

on a graph

a **hole** means the limit exists and the value does not; a filled dot elsewhere means they differ

average rate of change

the slope of a secant; the derivative is its limit

not tested

the epsilon–delta definition is **not** on the AP exam

“Ignore the point itself” is the whole idea. A function can be undefined exactly where its limit is clearest.

When substitution gives $\frac{0}{0}$

$\frac{0}{0}$ is an **indeterminate form** –
 it does **not** mean the limit fails.
 Rewrite into an **equivalent form**, then substitute.

The cancelled factor is **why the graph had a hole**: the two functions agree everywhere except at c .

Three standard moves

Factor and cancel :

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4$$

Multiply by the **conjugate** :

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1}{2}$$

Or use a **trig identity**.

Continuity over an interval, and repairing a hole

You rarely check point by point – whole families are continuous **on their domains**:

Polynomial, rational, power, **exponential**, **logarithmic** and **trigonometric** functions are continuous at every point of their domains.

So a rational function is continuous everywhere **except** where its denominator is zero.

Removing a discontinuity

If the limit exists at a hole, **redefine** the function there to equal the limit – the graph is repaired.

For a **piecewise** function at a boundary, solve for the **parameter** that makes the pieces **meet**:

$$\lim_{x \rightarrow c^-} f = \lim_{x \rightarrow c^+} f = f(c).$$

Exercises 2.1·2.3

2.1 State how to remove a removable discontinuity.

[1]

2.2 For $f(x) = \frac{x^2 - 4}{x - 2}$, find the value that removes the hole at 2.

[2]

2.3 State which kind of discontinuity can be removed.

[1]

Exercises 2.1·2.3

2.1 State how to remove a removable discontinuity.

[1]

2.1

- define $f(a)$ to equal the limit $\lim_{x \rightarrow a} f(x)$

2.2 For $f(x) = \frac{x^2 - 4}{x - 2}$, find the value that removes the hole at 2.

[2]

2.2

- the limit is 4, so set $f(2) = 4$

2.3 State which kind of discontinuity can be removed.

[1]

2.3

- a removable discontinuity (a hole)

Exercises 3.1-3.2

3.1 A removable discontinuity is removed by setting $f(a)$ equal to

[1]

☐

A 0

B the limit at a

C $f(a + 1)$

D ∞

3.2 Which discontinuity can be removed?

[1]

☐

A jump

B infinite

C removable

D none

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3.1 B

- the limit at a .

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[1]

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B infinite

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D none

3.2 C

- removable.

Exercise 3.3 (a)

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(a) Simplify f .

[1]

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[1]

(a)

- $x + 3$ (for $x \neq 3$)

Exercise 3.3 (b)

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(b) Find the limit at 3.

[1]

Exercise 3.3 (b)

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(b) Find the limit at 3.

[1]

(b)

- 6

Exercise 3.3 (c)

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(c) State $f(3)$ to remove the hole.

[1]

Exercise 3.3 (c)

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(c) State $f(3)$ to remove the hole.

[1]

(c)

- $f(3) = 6$

Exercise 3.4

3.4 (a) Find the value needed at 1 to extend $(x^2 - 1)/(x - 1)$ continuously. Show why. [2]

(b) Explain why the same single-point repair cannot remove a jump with left limit 2 and right limit 5. [2]

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3.4

(a) Cancellation gives $x + 1$ near 1, so assign the value 2.

(b) The one-sided limits differ, so no common two-sided limit exists for a point value to match.