

Exercise walk-through

1.12

Confirming Continuity over an Interval

AP Calculus AB

Three types of discontinuity

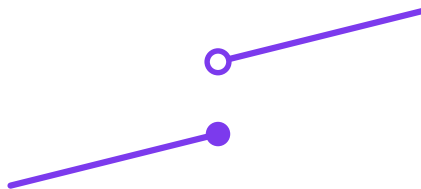
A function is **discontinuous**

at c when its graph **breaks** there:

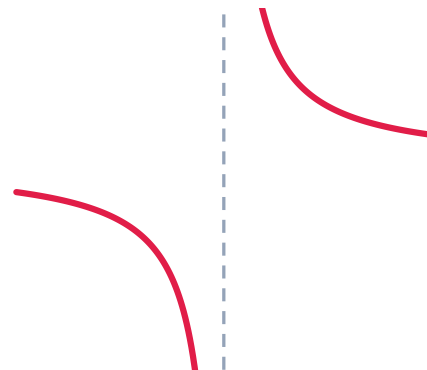
Removable
(hole)



Jump



Infinite
(asymptote)



f continuous at a iff $\lim_{x \rightarrow a} f(x) = f(a)$ (limit exists and equals the value)

Removable
(an asymptote).

(a hole) ■ **jump**

(sides disagree) ■ **infinite**

Classify the break, then decide whether it is repairable

Your turn, at $x = 2$

For each case, name the discontinuity and decide whether defining one point value can remove it. Give evidence about the nearby outputs.

$$A(x) = \frac{x^2 - 4}{x - 2}, \quad B(x) = \begin{cases} 1 & x < 2, \\ 3 & x \geq 2, \end{cases} \quad C(x) = \frac{1}{x - 2}.$$

For A, simplify for $x \neq 2$. For B and C, check each side separately.

A hole can be filled; a jump and an asymptote cannot

A: removable discontinuity

For $x \neq 2$, $A(x) = (x-2)(x+2)/(x-2) = x+2$. Hence $\lim_{x \rightarrow 2} A(x) = 4$. Defining $A(2) = 4$ removes the hole.

B: jump discontinuity

The one-sided limits are 1 and 3. They differ, so changing $B(2)$ cannot make them agree.

C: infinite discontinuity

As $x \rightarrow 2^-$, $C(x) \rightarrow -\infty$; as $x \rightarrow 2^+$, $C(x) \rightarrow +\infty$. There is no finite limiting value to supply at 2.

Continuity at a point – the three-part test

f is **continuous** at $x = c$ exactly when **all three** hold:

$f(c)$ **exists**

$\lim_{x \rightarrow c} f(x)$ **exists**

they **agree**: $\lim_{x \rightarrow c} f(x) = f(c)$

The point is there, the limit is there, and the two **agree**. If any one fails, f is discontinuous.

Learn it as a **checklist** – it is the backbone of nearly every continuity question.

Continuity over an interval, and repairing a hole

You rarely check point by point – whole families are continuous **on their domains**:

Polynomial, rational, power, **exponential**, **logarithmic** and **trigonometric** functions are continuous at every point of their domains.

So a rational function is continuous everywhere **except** where its denominator is zero.

Removing a discontinuity

If the limit exists at a hole, **redefine** the function there to equal the limit – the graph is repaired.

For a **piecewise** function at a boundary, solve for the **parameter** that makes the pieces **meet**:

$$\lim_{x \rightarrow c^-} f = \lim_{x \rightarrow c^+} f = f(c).$$

Exercises 2.1-2.3

2.1 State what continuity over an interval means.

[1]

2.2 State where a polynomial is continuous.

[1]

2.3 State where a rational function may be discontinuous.

[2]

Exercises 2.1-2.3

2.1 State what continuity over an interval means.

[1]

2.1

- it is continuous at every point of the interval
-

2.2 State where a polynomial is continuous.

[1]

2.2

- everywhere (for all real x)
-

2.3 State where a rational function may be discontinuous.

[2]

2.3

- where its denominator equals 0

Exercises 3.1-3.2

3.1 A function is continuous on an interval if it is continuous

[1]

☐

A at one point

B at every point of the interval

C at the endpoints only

D nowhere

☐

3.2 Polynomials are continuous

[1]

A nowhere

B everywhere

C only at 0

D only for $x > 0$

Exercises 3.1-3.2

3.1 A function is continuous on an interval if it is continuous

[1]

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B at every point of the interval

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3.1 B

- at every point of the interval.

3.2 Polynomials are continuous

[1]

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A nowhere

B everywhere

C only at 0

D only for $x > 0$

3.2 B

- everywhere.

Exercise 3.3 (a)

3.3 $f(x) = \frac{1}{x-3}.$

(a) Where is it discontinuous?

[1]

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3.3 $f(x) = \frac{1}{x-3}$.

(a) Where is it discontinuous?

[1]

(a)

- $x = 3$

Exercise 3.3 (b)

3.3 $f(x) = \frac{1}{x-3}.$

(b) Is it continuous on $[0, 2]$?

[1]

Exercise 3.3 (b)

3.3 $f(x) = \frac{1}{x-3}.$

(b) Is it continuous on $[0, 2]$?

[1]

(b)

- yes (3 is not in $[0, 2]$)

Exercise 3.3 (c)

3.3 $f(x) = \frac{1}{x-3}.$

(c) Is it continuous on $[0, 4]$?

[1]

Exercise 3.3 (c)

3.3 $f(x) = \frac{1}{x-3}.$

(c) Is it continuous on $[0, 4]$?

[1]

(c)

- no (3 is in $[0, 4]$)

Exercise 3.4

3.4 For $g(x) = \ln(x - 2)/(x - 5)$, (a) identify both domain restrictions. [2]

(b) State the maximal intervals of continuity and explain why they are separate. [2]

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3.4

(a) $x > 2$ for the logarithm and $x \neq 5$ for the denominator.

(b) $(2, 5)$ and $(5, \infty)$; the missing input 5 separates them.