Exercise walk-through

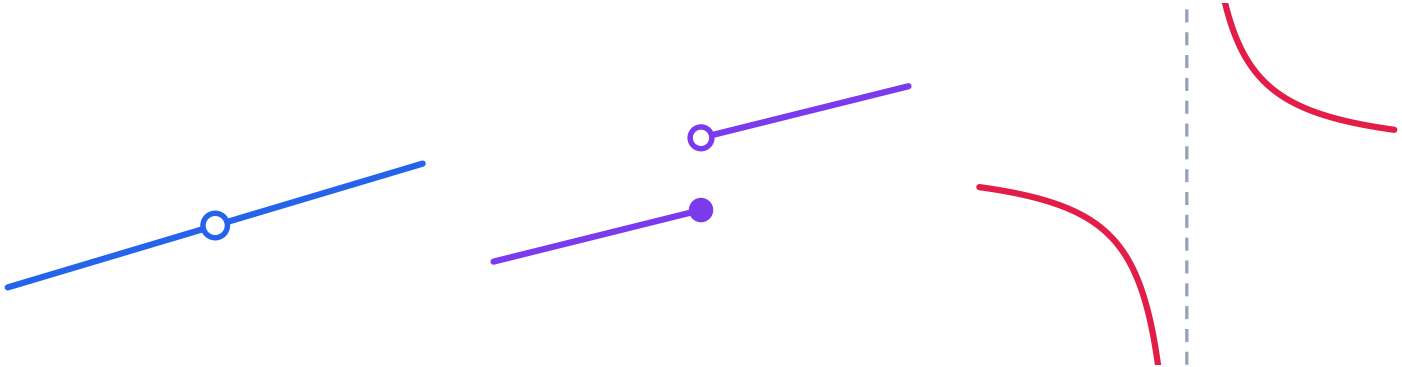
1.11
Defining Continuity at a Point

AP Calculus AB

Three types of discontinuity

A function is **discontinuous** at c when its graph breaks there:

Removable (hole) | Jump | Infinite (asymptote)



f continuous at a iff $\lim_{x \to a} f(x) = f(a)$ (limit exists and equals the value)

Removable (a hole) ▪ **jump** (sides disagree) ▪ **infinite** (an asymptote).

Classify the break, then decide whether it is repairable

Your turn, at $x = 2$

For each case, name the discontinuity and decide whether defining one point value can remove it. Give evidence about the nearby outputs.

$$A(x) = \frac{x^2 - 4}{x - 2}, \quad B(x) = \begin{cases} 1 & x < 2, \\ 3 & x \geq 2, \end{cases} \quad C(x) = \frac{1}{x - 2}.$$

For A, simplify for $x \neq 2$. For B and C, check each side separately.

A hole can be filled; a jump and an asymptote cannot

A: removable discontinuity

For $x \neq 2$, $A(x) = (x-2)(x+2)/(x-2) = x+2$. Hence $\lim_{x\to 2} A(x) = 4$. Defining $A(2) = 4$ removes the hole.

B: jump discontinuity

The one-sided limits are 1 and 3. They differ, so changing $B(2)$ cannot make them agree.

C: infinite discontinuity

As $x \to 2^-$, $C(x) \to -\infty$; as $x \to 2^+$, $C(x) \to +\infty$. There is no finite limiting value to supply at 2.

Continuity at a point – the three-part test

f is **continuous** at $x = c$ exactly when all three hold:

$f(c)$ **exists**

$\lim_{x \to c} f(x)$ **exists**

they **agree**: $\lim_{x \to c} f(x) = f(c)$

> The point is there, the limit is there, and the two agree. If any one fails, f is discontinuous.

Learn it as a checklist – it is the backbone of nearly every continuity question.

Continuity over an interval, and repairing a hole

You rarely check point by point – whole families are continuous on their domains:

> Polynomial, rational, power, **exponential**, **logarithmic** and **trigonometric** functions are continuous at every point of their domains.

So a rational function is continuous everywhere except where its denominator is zero.

Removing a discontinuity

If the limit exists at a hole, redefine the function there to equal the limit – the graph is repaired.

For a **piecewise** function at a boundary, solve for the **parameter** that makes the pieces meet:

$$\lim_{x \to c^-} f = \lim_{x \to c^+} f = f(c).$$

Exercises 2.1·2.3

2.1 State the three conditions for continuity at a. [1]

2.2 State the single equation summarizing continuity at a. [1]

2.3 If $\lim_{x \to 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

Exercises 2.1·2.3

2.1 State the three conditions for continuity at a. [1]

2.1

- $f(a)$ is defined; the limit exists; the limit equals $f(a)$

2.2 State the single equation summarizing continuity at a. [1]

2.2

- $\lim_{x \to a} f(x) = f(a)$

2.3 If $\lim_{x \to 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

2.3

- yes – the limit exists, $f(2)$ is defined, and they are equal

Exercises 3.1·3.2

3.1 f is continuous at a if and only if [1]

- ☐ **A** $f(a)$ is defined
- **B** $\lim_{x \to a} f(x) = f(a)$
- **C** the limit exists only
- **D** f is linear

3.2 If $\lim_{x \to a} f(x)$ exists but $\neq f(a)$, then f is [1]

- ☐ **A** continuous at a
- **B** discontinuous at a
- **C** linear
- **D** undefined everywhere

Exercises 3.1·3.2

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3.1 B

- $\lim_{x\to a} f(x) = f(a)$.

3.2 If $\lim_{x\to a} f(x)$ exists but $\neq f(a)$, then f is [1]

- ☐ **A** continuous at a
- **B** discontinuous at a
- **C** linear
- **D** undefined everywhere

3.2 B

- discontinuous at a.

Exercise 3.3 (a)

3.3 $f(2) = 5$ and $\lim_{x \to 2} f(x) = 5$.

(a) Is $f(2)$ defined? [1]

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(a)

- yes

Exercise 3.3 (b)

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(b) Does the limit exist? [1]

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(b)

- yes

Exercise 3.3 (c)

3.3 $f(2) = 5$ and $\lim_{x \to 2} f(x) = 5$.

(c) Is f continuous at 2? [1]

Exercise 3.3 (c)

3.3 $f(2) = 5$ and $\lim_{x \to 2} f(x) = 5$.

(c) Is f continuous at 2? [1]

(c)

- yes

Exercise 3.4

3.4 Let $g(x) = x^2 - x + 1$ for $x \neq 2$ and $g(2) = k$. (a) Find the limit at 2 and the value of k required for continuity. [2]

(b) State the three continuity conditions and explain why your assignment meets them. [2]

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3.4

(a) Limit $= 4 - 2 + 1 = 3$; $k = 3$.

(b) The point value and limit both exist and are equal at 3.