

Exercise walk-through

1.10

Exploring Types of Discontinuities

AP Calculus AB

Three types of discontinuity

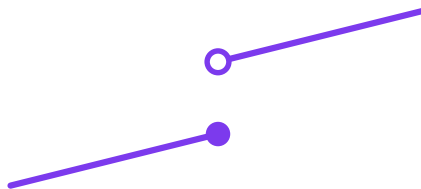
A function is **discontinuous**

at c when its graph **breaks** there:

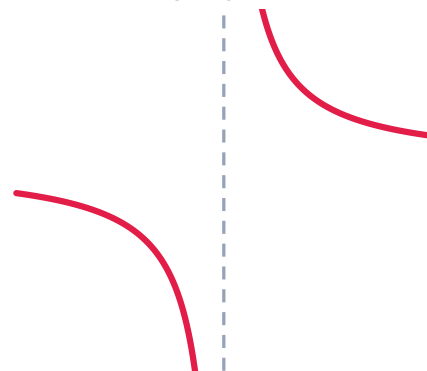
Removable
(hole)



Jump



Infinite
(asymptote)



f continuous at a iff $\lim_{x \rightarrow a} f(x) = f(a)$ (limit exists and equals the value)

Removable
(an asymptote).

(a hole) ■ **jump**

(sides disagree) ■ **infinite**

Classify the break, then decide whether it is repairable

Your turn, at $x = 2$

For each case, name the discontinuity and decide whether defining one point value can remove it. Give evidence about the nearby outputs.

$$A(x) = \frac{x^2 - 4}{x - 2}, \quad B(x) = \begin{cases} 1 & x < 2, \\ 3 & x \geq 2, \end{cases} \quad C(x) = \frac{1}{x - 2}.$$

For A, simplify for $x \neq 2$. For B and C, check each side separately.

A hole can be filled; a jump and an asymptote cannot

A: removable discontinuity

For $x \neq 2$, $A(x) = (x-2)(x+2)/(x-2) = x+2$. Hence $\lim_{x \rightarrow 2} A(x) = 4$. Defining $A(2) = 4$ removes the hole.

B: jump discontinuity

The one-sided limits are 1 and 3. They differ, so changing $B(2)$ cannot make them agree.

C: infinite discontinuity

As $x \rightarrow 2^-$, $C(x) \rightarrow -\infty$; as $x \rightarrow 2^+$, $C(x) \rightarrow +\infty$. There is no finite limiting value to supply at 2.

Exercises 2.1-2.3

2.1 Name the three types of discontinuity.

[1]

2.2 State which type is a “hole”.

[1]

2.3 State which type occurs at a vertical asymptote.

[2]

Exercises 2.1-2.3

2.1 Name the three types of discontinuity. [1]

2.1

- removable, jump, and infinite
-

2.2 State which type is a “hole”. [1]

2.2

- removable
-

2.3 State which type occurs at a vertical asymptote. [2]

2.3

- infinite

Exercises 3.1-3.2

3.1 A discontinuity where the left and right limits differ is a

[1]

☐

A removable discontinuity

B jump discontinuity

C infinite discontinuity

D point of continuity

3.2 A hole in a graph is a

[1]

☐

A jump discontinuity

B removable discontinuity

C infinite discontinuity

D point of continuity

Exercises 3.1-3.2

3.1 A discontinuity where the left and right limits differ is a

[1]

☐

A removable discontinuity

B jump discontinuity

C infinite discontinuity

D point of continuity

3.1 B

- jump discontinuity.

3.2 A hole in a graph is a

[1]

☐

A jump discontinuity

B removable discontinuity

C infinite discontinuity

D point of continuity

3.2 B

- removable discontinuity.

Exercise 3.3 (a)

3.3 Classify each.

(a) A vertical asymptote.

[1]

Exercise 3.3 (a)

3.3 Classify each.

(a) A vertical asymptote.

[1]

(a)

- infinite

Exercise 3.3 (b)

3.3 Classify each.

(b) A single missing point.

[1]

Exercise 3.3 (b)

3.3 Classify each.

(b) A single missing point.

[1]

(b)

- removable

Exercise 3.3 (c)

3.3 Classify each.

(c) A step where the graph jumps.

[1]

Exercise 3.3 (c)

3.3 Classify each.

(c) A step where the graph jumps.

[1]

(c)

- jump

Exercise 3.4

3.4 (a) For $f(x) = (x^2 - 4)/(x - 2)$, identify the discontinuity at 2 and give a point value that removes it. [2]

(b) A function approaches 1 from the left of 3 and 4 from the right. Classify the discontinuity and explain whether changing $f(3)$ can remove it. [2]

Exercise 3.4

3.4 (a) For $f(x) = (x^2 - 4)/(x - 2)$, identify the discontinuity at 2 and give a point value that removes it. [2]

(b) A function approaches 1 from the left of 3 and 4 from the right. Classify the discontinuity and explain whether changing $f(3)$ can remove it. [2]

3.4

(a) Removable; define $f(2) = 4$.

(b) Jump discontinuity; no single point value can make the unequal one-sided limits agree.