HOLIDAY HOMEWORK 假期作业

AP Calculus AB

Exercise sheets and past-paper practice, topic by topic.
每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

Contents 目录

AP CALCULUS AB • HOLIDAY HOMEWORK 假期作业

1 What a limit is – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
limit	极限	

Recall

You have looked at graphs and how they behave:

- You can trace a curve and see where it is heading.
- Some graphs get closer and closer to a value without quite reaching it.
- You know how to put a number into a formula.

This sheet introduces the **limit** – the first big idea of calculus. Each idea has a worked example.

New ideas

1 What a limit is

A **limit** 极限 is the value a function **approaches** as x gets close to some point. We write it

$$\lim_{x\to a} f(x).$$

It asks: as x heads toward a, what does $f(x)$ head toward?

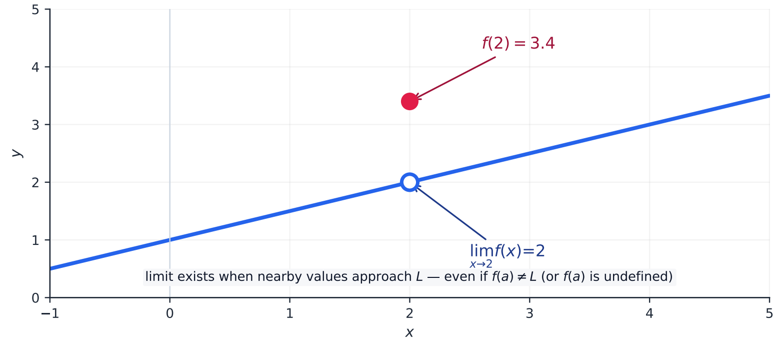
2 Finding a limit

If the function has no break at a, you can just substitute.

Worked example. $\lim_{x\to 2}(x^2+1) = 2^2+1 = 5$.

3 A hole in the graph

A limit can exist even if the function has a **hole** at that point – the graph still heads toward a value from both sides.



4 From both sides

The limit exists only if the function heads toward the **same** value from the left and from the right.

Watch out: the limit is about what $f(x)$ **approaches**, not necessarily the value at that exact point. A function can approach 5 even if, right at that point, it has a hole or a different value.

Practice

Try these in order. The methods are all above.

2.1 In words, state what $\lim_{x\to a} f(x)$ means. [2]

2.2 Find $\lim_{x\to 3}(x+4)$. [2]

2.3 Find $\lim_{x \to 2}(x^2 - 1)$. [2]

2.4 State whether a limit can exist at a point where the graph has a hole. [1]

2.5 State the condition on the left and right sides for a limit to exist. [2]

6. The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x.

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Part A

Find $\lim\limits_{x \to 2} \dfrac{f(x)}{x}$, or state that the limit does not exist.

Part B

Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.

Part C

Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.

Part D

Let k be the function defined by $k(x) = \int_0^x t^2 f(t)\,dt$.

i. Find $k'(x)$.

ii. Find $k''(3)$. Show the work that leads to your answer.

STOP

END OF EXAM

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{\text{[illegible]}}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$.

(b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

STOP

END OF EXAM

1 Continuity and asymptotes – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
continuous	连续			
vertical asymptote	渐近线			

Recall

In Part A you met limits. Now use them to describe how smooth a graph is:

- Some graphs you can draw without lifting your pen.
- Others have jumps, holes, or shoot off to infinity.

Each idea has a worked example before the Practice.

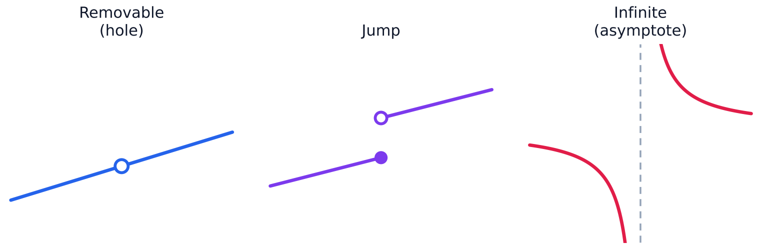
New ideas

1 Continuity

A function is **continuous** 连续 at a point if it has no break there – no hole, jump, or gap. A continuous graph can be drawn without lifting your pen.

2 Discontinuities

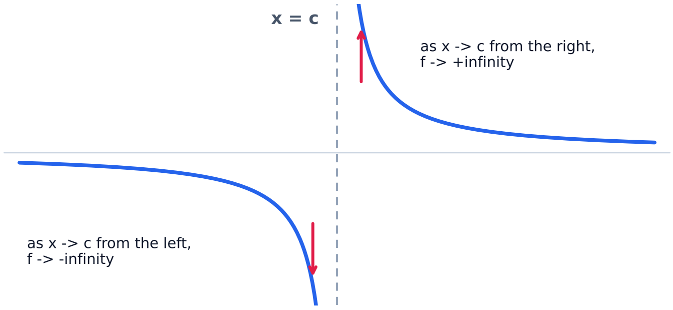
Where a graph breaks, it is **discontinuous**. The main kinds are a **hole**, a **jump**, and an **asymptote**.



f continuous at *a* iff $\lim_{x\to a} f(x)=f(a)$ (limit exists and equals the value)

■ 3 Vertical asymptotes

A **vertical asymptote** 渐近线 is a line the graph shoots up (or down) toward, without ever touching – it happens where the function blows up to infinity (like dividing by zero).



vertical asymptote $x = c$ when $\lim_{x\to c^+} f(x) = \pm\infty$ or $\lim_{x\to c^-} f(x) = \pm\infty$

■ 4 Horizontal asymptotes

A **horizontal asymptote** is a value the graph levels off toward as x gets very large (or very negative).

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one – near a vertical asymptote the function races off to $\pm\infty$ instead.

Practice

Try these in order. The methods are all above.

2.1 State what it means for a function to be continuous at a point. [2]

2.2 Name three kinds of discontinuity. [3]

2.3 State what happens to a function near a vertical asymptote. [2]

2.4 State what a horizontal asymptote describes. [2]

2.5 For $f(x) = \dfrac{1}{x-3}$, state where the vertical asymptote is. [1]

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{[illegible]}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$.

(b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

STOP

END OF EXAM

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t, for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

2 The derivative as a slope – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
derivative	导数	
tangent	切线	

Recall

You already know about the gradient (slope) of a line:

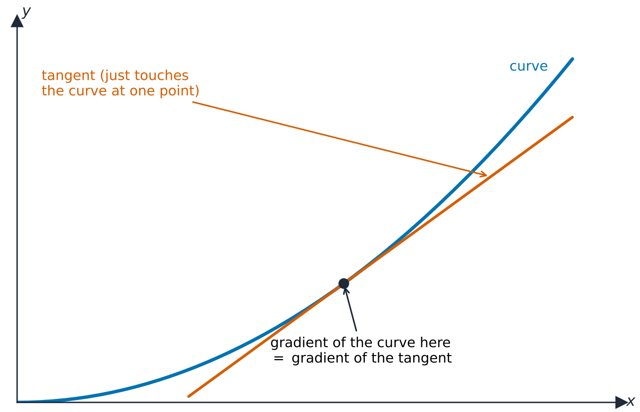
- A steep line has a big gradient; a flat one has zero.
- Gradient = rise over run.
- But a curve's steepness keeps changing along it.

This sheet introduces the **derivative** – the slope of a curve. Each idea has a worked example.

New ideas

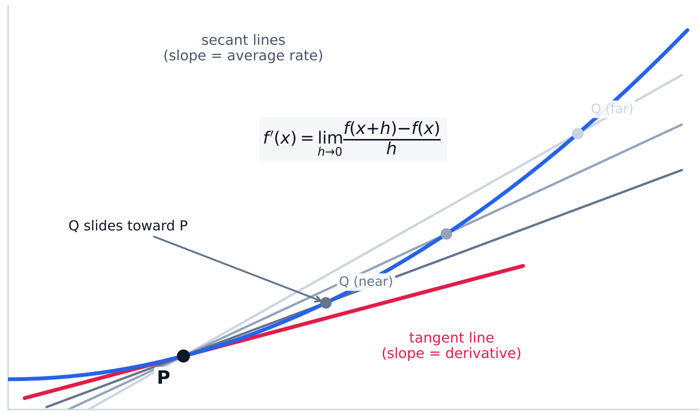
1 The slope of a curve

The **derivative** 导数 of a function at a point is the **slope of the curve** there – how steep it is at that exact spot.



2 From secant to tangent

Take two points on the curve and join them – the **secant line**. Its slope is the **average** rate of change. As the two points move closer together, the secant becomes the **tangent** 切线 line, whose slope is the derivative.



3 Notation

The derivative of f is written $f'(x)$ or $\dfrac{dy}{dx}$.

■ 4 A rate of change

The derivative is a **rate of change**: how fast the output changes as the input changes. For position over time, that rate is the velocity.

Watch out: the derivative is the slope of the **tangent** (at a single point), not the secant (between two points). The secant gives the **average** rate of change; the derivative gives the rate at one instant.

Practice

Try these in order. The methods are all above.

2.1 State what the derivative of a function tells you about its graph. [2]

2.2 State the difference between a secant line and a tangent line. [2]

2.3 State the two common notations for the derivative. [2]

2.4 A line has gradient 3. State the derivative of that line at every point. [1]

2.5 For an object's position over time, state what its derivative represents. [1]

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

   (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

   (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

   (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

   (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

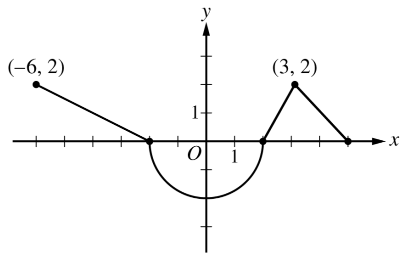
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

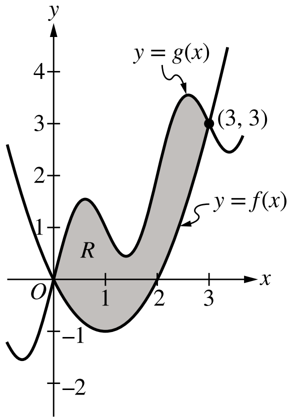
(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

2. The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

A. Find the area of R. Show the setup for your calculations.

B. Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

D. It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

END OF PART A

2 The power rule – Part 2

Name: ____________ Class: __________ Date: ____________

Recall

In Part A you learned that the derivative is the slope of a curve. Now compute it quickly with a rule:

- Finding a slope from scratch each time would be slow.
- A simple rule handles powers of x.

Each idea has a worked example before the Practice.

New ideas

■ 1 The power rule

To differentiate a power of x, multiply by the power and lower the power by one:

$$\frac{d}{dx}x^n = n\,x^{n-1}.$$

Worked example. $\frac{d}{dx}x^3 = 3x^2$, and $\frac{d}{dx}x^5 = 5x^4$.

■ 2 Constants

The derivative of a **constant** is 0 (a flat line has zero slope). A constant multiplier stays: $\frac{d}{dx}(5x^2) = 10x$.

■ 3 Sums

Differentiate a sum term by term.

Worked example. $\frac{d}{dx}(x^2 + 3x) = 2x + 3$.

■ 4 Reading the result

The derivative is itself a function – put in an x value to get the slope there.

Worked example. For $f(x) = x^2$, $f'(x) = 2x$, so the slope at $x = 3$ is $f'(3) = 6$.

Watch out: the power rule **lowers** the power by one and **multiplies** by the old power. A common slip is to forget the multiply – $\frac{d}{dx}x^4$ is $4x^3$, not x^3.

Practice

Try these in order. The methods are all above.

2.1 Find $\dfrac{d}{dx}x^4$. [2]

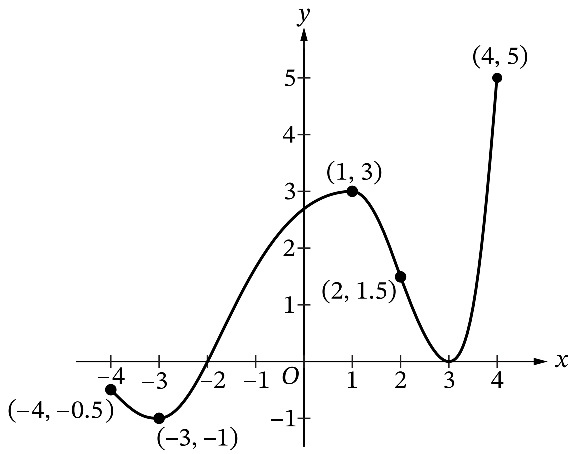
2.2 Find $\dfrac{d}{dx}x^7$. [1]

2.3 Find the derivative of $f(x) = x^2 + 5x$. [2]

2.4 Find the derivative of $f(x) = 3x^2$. [2]

2.5 For $f(x) = x^2$, find the slope of the curve at $x = 4$. [2]

4. Let f be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f', the derivative of f, is shown.



Graph of f'

Part A

For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.

Part B

Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

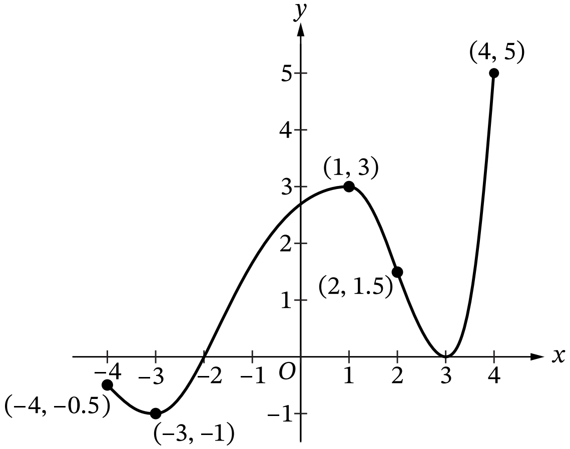
Part C

For $-4 \le x \le 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.

Part D

For $-4 \le x \le 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.

4. Let f be a twice-differentiable function on the closed interval $[-4,4]$ with $f(2)=3$. The graph of f', the derivative of f, is shown.



Graph of f'

Part A

For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.

Part B

Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

Part C

For $-4 \le x \le 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.

Part D

For $-4 \le x \le 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.

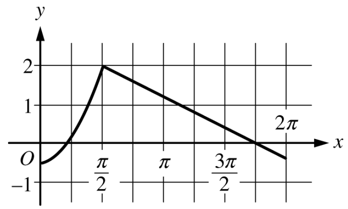
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\, dx$.

STOP

END OF EXAM

AP CALCULUS AB • HOLIDAY HOMEWORK 假期作业

3 The chain rule – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
chain rule	链式法则	

Recall

In the last topic you used the power rule to differentiate powers of x:

- $\frac{d}{dx}x^3 = 3x^2$.
- But what about something like $(x^2+1)^3$ – a function **inside** another function?

This sheet gives the rule for that: the chain rule. Each idea has a worked example.

New ideas

1 Composite functions

A **composite function** is one function tucked **inside** another, like $(x^2+1)^3$: the "inside" is x^2+1 and the "outside" is "cube it".

2 The chain rule

To differentiate a composite function, use the **chain rule** 链式法则:

> differentiate the **outside** (keeping the inside as it is), then **multiply** by the derivative of the inside.

Worked example. For $(x^2+1)^3$: the outside gives $3(x^2+1)^2$, and the inside x^2+1 has derivative $2x$, so

$$\frac{d}{dx}(x^2+1)^3 = 3(x^2+1)^2 \cdot 2x.$$

3 Another example

Worked example. For $(3x+1)^4$: outside $\to 4(3x+1)^3$; inside $3x+1$ has derivative 3; so the answer is $4(3x+1)^3 \cdot 3 = 12(3x+1)^3$.

4 Why "chain"?

The rule links the rates together in a chain: how fast the outside changes with the inside, times how fast the inside changes with x.

Watch out: don't forget to multiply by the derivative of the **inside**. Writing just $3(x^2+1)^2$ (and forgetting the $\times 2x$) is the most common chain-rule mistake.

Practice

Try these in order. The methods are all above.

2.1 State, in words, what the chain rule tells you to do. [2]

2.2 For $(x^2+1)^3$, state the "inside" function and its derivative. [2]

2.3 Differentiate $(2x+5)^3$. [3]

2.4 Differentiate $(x^2+4)^2$. [3]

2.5 State the most common mistake when using the chain rule. [1]

5. Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y\sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y\cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\dfrac{dy}{dx} = \dfrac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\dfrac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

STOP

END OF EXAM

3 The second derivative and inverse functions – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
second derivative	二阶导数	____ ____ ____

Recall

In Part A you met the chain rule. Now two more ideas:

- You can differentiate a derivative again.
- Some functions are the “reverse” of others (inverses).

Each idea has a worked example before the Practice.

New ideas

1 The second derivative

If you differentiate a function and then differentiate **again**, you get the **second derivative** 二阶导数, written $f''(x)$.

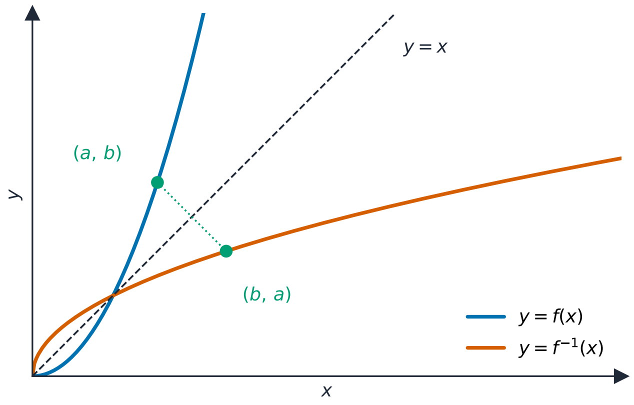
Worked example. For $f(x) = x^3$: the first derivative is $f'(x) = 3x^2$, and differentiating again gives $f''(x) = 6x$.

2 What it means

The first derivative is the **rate of change** (like velocity); the second derivative is the **rate of change of that rate** (like acceleration). It tells you whether the slope is increasing or decreasing.

3 Inverse functions

An **inverse function** reverses a function. Its graph is the reflection of the original in the line $y = x$.



4 Building up

Worked example. For $f(x) = x^2 + 3x$: $f'(x) = 2x + 3$, and $f''(x) = 2$ (a constant, so the slope changes at a steady rate).

Watch out: the second derivative is found by differentiating **twice** – first get $f'(x)$, then differentiate that. It is the derivative of the derivative, not the square of the derivative.

Practice

Try these in order. The methods are all above.

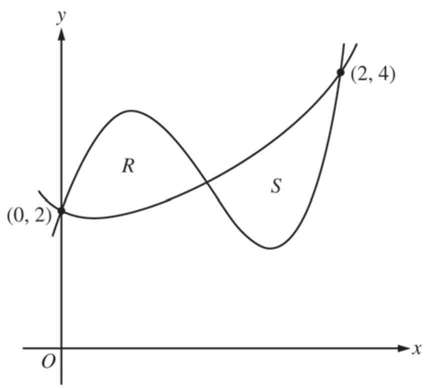
2.1 State what the second derivative is. [1]

2.2 For $f(x) = x^4$, find $f'(x)$ and then $f''(x)$. [3]

2.3 For position over time, state what the first and second derivatives represent. [2]

2.4 State how the graph of an inverse function relates to the original. [2]

2.5 For $f(x) = x^2$, find $f''(x)$. [2]



2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

END OF PART A OF SECTION II

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\dfrac{dy}{dx} = \dfrac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\dfrac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

STOP

END OF EXAM

4 Motion - velocity and acceleration – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
velocity	速度	
acceleration	加速度	

Recall

The derivative is a rate of change – and motion is all about rates:

- Velocity is how fast your position changes.
- Acceleration is how fast your velocity changes.
- Calculus connects all three.

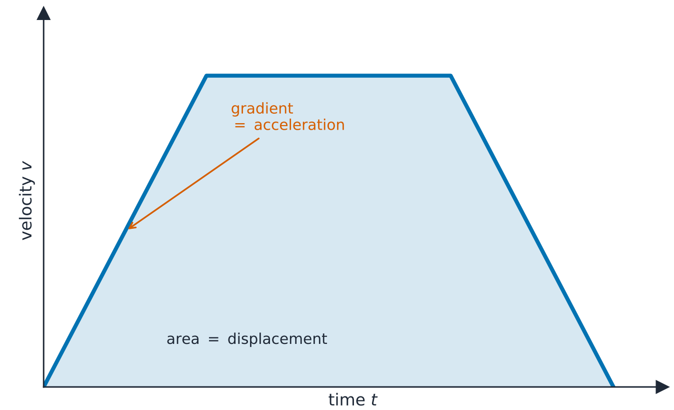
This sheet lines up motion with derivatives. Each idea has a worked example.

New ideas

1 Position, velocity, acceleration

For an object moving along a line:

- **velocity** 速度 is the derivative of position (how fast position changes),
- **acceleration** 加速度 is the derivative of velocity (how fast velocity changes).



2 From position to velocity

Worked example. If position is $s(t) = t^2$, then velocity is $s'(t) = 2t$. At $t = 3$ the velocity is $2 \times 3 = 6$.

3 From velocity to acceleration

Worked example. With velocity $v(t) = 2t$, the acceleration is $v'(t) = 2$ – a constant.

4 Direction from the sign

A **positive** velocity means moving in the positive direction; a **negative** velocity means moving the other way. The object is momentarily at rest when velocity is 0.

Watch out: velocity is the derivative of **position**, and acceleration is the derivative of **velocity** – so acceleration is the derivative of position **twice** (the second derivative).

Practice

Try these in order. The methods are all above.

2.1 State how velocity is related to position, and how acceleration is related to velocity.[2]

2.2 Position is $s(t) = t^2$. Find the velocity $s'(t)$. [2]

2.3 Using that velocity, find the velocity at $t = 5$. [2]

2.4 For velocity $v(t) = 3t$, find the acceleration. [2]

2.5 State what a velocity of 0 tells you about the object's motion. [1]

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

   (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

   (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

   (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

   (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

5. Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.
B. During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.
C. It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.
D. Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \ge 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4 Related rates and linear approximation – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Related rates	相关变化率	______ ______ ______

Recall

In Part A you used derivatives for motion. Two more uses:

- When one thing changes, a connected thing changes too.
- A tangent line can estimate values near a known point.

Each idea has a worked example before the Practice.

New ideas

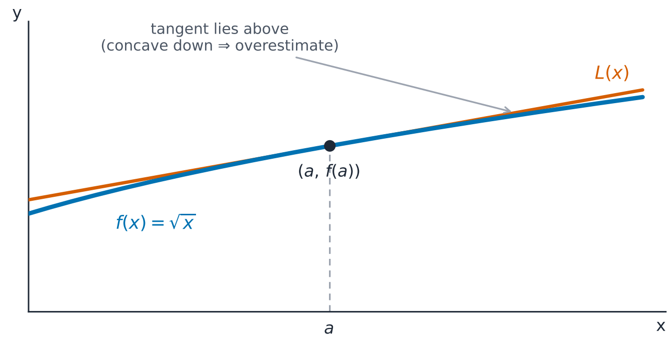
1 Related rates

Related rates 相关变化率 problems appear when two quantities change together over time. If you know how fast one changes, you can find how fast the other does.

Worked example. A square's side grows at 2 cm/s. Since area $A = s^2$, the area's rate of change is $\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$. When $s = 5$, $\frac{dA}{dt} = 2(5)(2) = 20$ cm^2/s.

2 Linear approximation

The tangent line at a point stays close to the curve nearby, so it gives a quick **estimate** of the function's value.



$f(x) \approx L(x) = f(a) + f'(a)(x-a)$ near $x = a$

3 Using the tangent

Near a known point, $f(x) \approx f(a) + f'(a)(x-a)$ – follow the tangent line a small step.

Worked example. For $f(x) = x^2$ near $x = 3$: $f(3) = 9$ and $f'(3) = 6$, so $f(3.1) \approx 9 + 6(0.1) = 9.6$ (the true value is 9.61).

4 Why it works

Because a smooth curve looks almost straight when you zoom in, the tangent is an excellent approximation close to the point.

Watch out: a tangent-line estimate is only good **close** to the point of contact. Far from it, the curve bends away from the tangent and the approximation gets worse.

Practice

Try these in order. The methods are all above.

2.1 State what a related-rates problem involves. [2]

2.2 A square's side grows at 3 cm/s. Using $\frac{dA}{dt} = 2s\frac{ds}{dt}$, find the rate the area grows when $s = 4$. [3]

2.3 State what a linear approximation uses to estimate a function's value. [1]

2.4 For $f(x) = x^2$ near $x = 2$ ($f(2) = 4$, $f'(2) = 4$), estimate $f(2.1)$. [3]

2.5 State when a tangent-line approximation stops being accurate. [1]

6. Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.

B. There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

C. For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.

D. A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

STOP

END OF EXAM

6. Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.

B. There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

C. For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.

D. A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

STOP

END OF EXAM

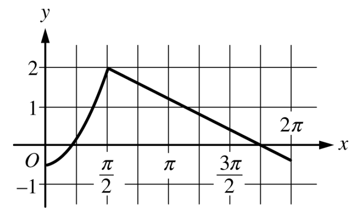
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

5 Increasing, decreasing, and turning points – Part 1

Name: ____________ Class: ________ Date: __________

Recall

The derivative gives the slope of a curve – and the slope tells you a lot:

- Where a graph goes uphill, the slope is positive.
- At the very top of a hill, the graph is momentarily flat.
- Businesses want to find the "best" (maximum or minimum) point.

This sheet uses the derivative to find high and low points. Each idea has a worked example.

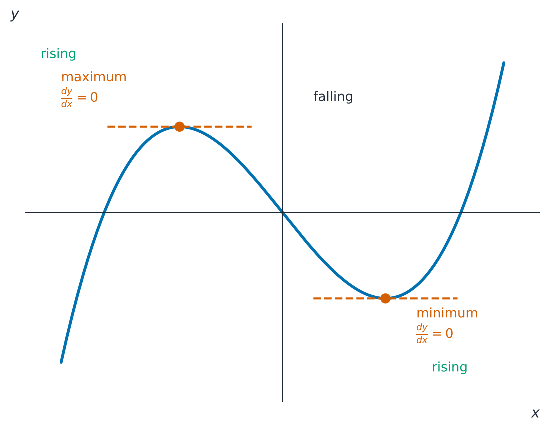
New ideas

1 Increasing and decreasing

- Where the derivative $f'(x) > 0$, the function is **increasing** (going uphill).
- Where $f'(x) < 0$, it is **decreasing** (going downhill).

2 Stationary points

Where $f'(x) = 0$, the curve is momentarily **flat** – a **stationary point** (a peak or a valley).



3 Finding a maximum or minimum

To find high and low points: differentiate, set $f'(x) = 0$, and solve for x.

Worked example. For $f(x) = x^2 - 4x$: $f'(x) = 2x - 4$. Setting $2x - 4 = 0$ gives $x = 2$ – the minimum point of this parabola.

4 Peak or valley?

Check the slope just before and after: if it changes from positive to negative, it is a **maximum**; from negative to positive, a **minimum**.

Watch out: a stationary point ($f' = 0$) can be a maximum **or** a minimum (or neither). Finding where $f'(x) = 0$ locates the point; you still have to check which kind it is.

Practice

Try these in order. The methods are all above.

2.1 State what the sign of $f'(x)$ tells you about the graph. [2]

2.2 State what is true about the derivative at a stationary point. [1]

2.3 For $f(x) = x^2 - 6x$, find $f'(x)$ and the x-value where it is zero. [3]

2.4 For $f(x) = x^2 + 2x$, find the x-value of the stationary point. [2]

2.5 State how you decide whether a stationary point is a maximum or a minimum. [2]

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

(d) A line forms whenever $A(t) \ge 400$. The number of vehicles in line at time t, for $a \le t \le 4$, is given by $N(t) = \int_a^t (A(x) - 400)\, dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \le t \le 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5 Concavity and the shape of a curve – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you found peaks and valleys with the first derivative. Now the **second** derivative describes the curve's bend:

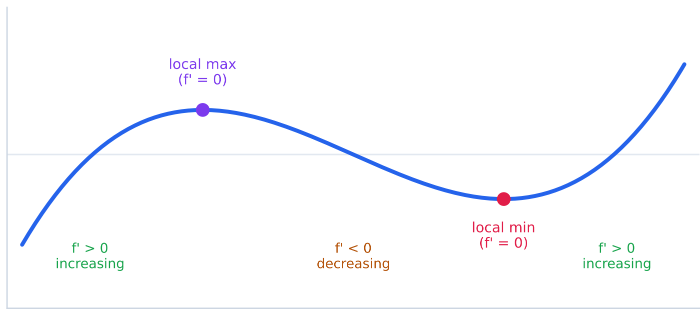
- A curve can bend upward (like a smile) or downward (like a frown).
- The second derivative tells you which.

Each idea has a worked example before the Practice.

New ideas

1 Concavity

The way a curve bends is its **concavity**:



$f' > 0 \Rightarrow$ increasing · $f' < 0 \Rightarrow$ decreasing · $f' = 0$ candidates for extrema

- $f''(x) > 0$: **concave up** (a smile / valley shape),
- $f''(x) < 0$: **concave down** (a frown / hill shape).

■ **2 Points of inflection**

Where the curve **changes** its bend (from concave up to concave down or the reverse) is a **point of inflection** – there $f''(x) = 0$.

■ **3 The second-derivative test**

At a stationary point ($f' = 0$): if $f'' > 0$ it is a **minimum** (valley), and if $f'' < 0$ it is a **maximum** (hill).

Worked example. For $f(x) = x^2 - 4x$: $f'(x) = 2x - 4 = 0$ at $x = 2$, and $f''(x) = 2 > 0$, so $x = 2$ is a **minimum**.

■ **4 Putting it together**

The first derivative finds where the peaks and valleys are; the second derivative tells you which is which and how the curve bends.

Watch out: concave up is a "smile" (holds water) and concave down is a "frown" (spills water). A concave-up curve can still be going downhill – concavity is about the **bend**, not the slope.

Practice

Try these in order. The methods are all above.

2.1 State what $f''(x) > 0$ and $f''(x) < 0$ tell you about a curve's shape. [2]

2.2 State what a point of inflection is. [2]

2.3 For $f(x) = x^2 + 6x$: find $f'(x)$, the stationary point, and use $f''(x)$ to say if it is a max or min. [3]

2.4 State the second-derivative test for a minimum. [1]

2.5 Explain why "concave up" does not always mean the curve is going uphill. [2]

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

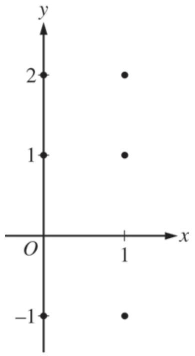
(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

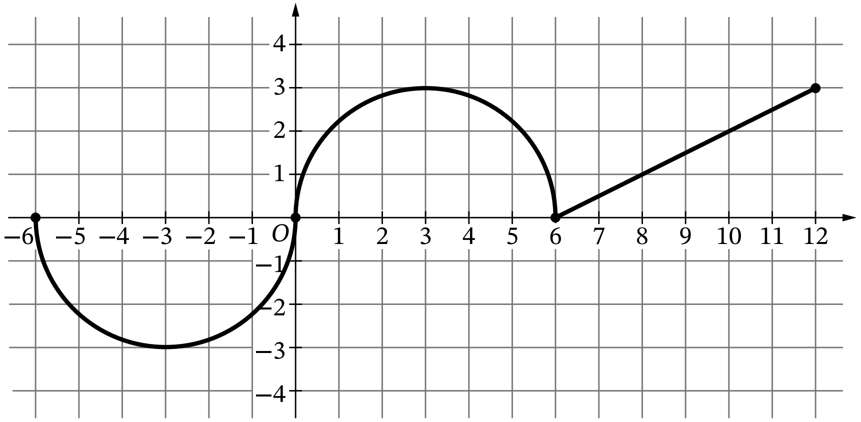


(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

4. The continuous function f is defined on the closed interval $-6 \le x \le 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t)\,dt$.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer.

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

STOP

END OF EXAM

AP CALCULUS AB • HOLIDAY HOMEWORK 假期作业

6 The area under a curve – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Riemann sum	黎曼和	

Recall

You can find the area of simple shapes:

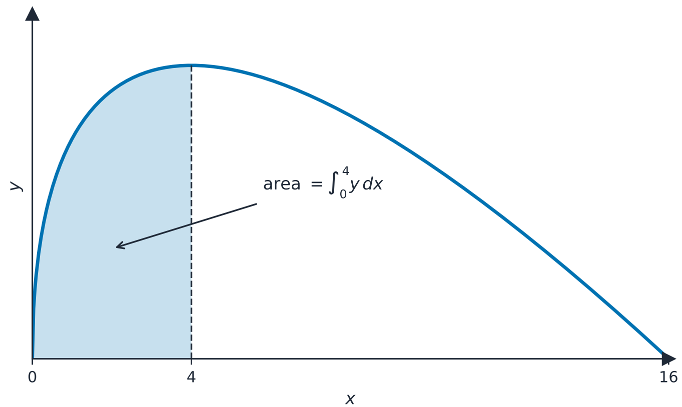
- The area of a rectangle is width × height.
- But how do you find the area under a **curve**, which has no straight edges?

This sheet introduces the idea behind **integration**. Each idea has a worked example.

New ideas

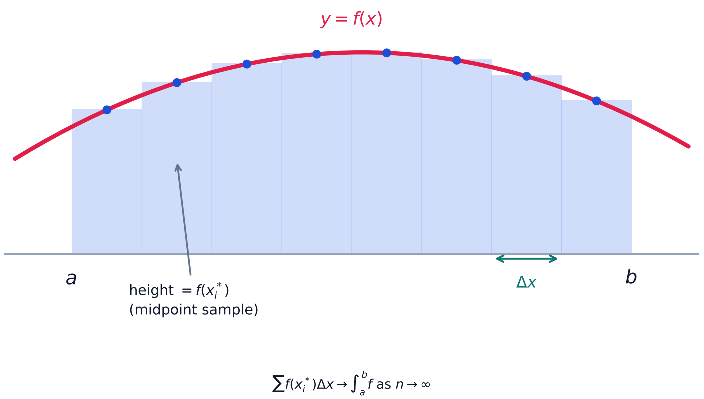
1 Area under a curve

The **area under a curve** (between the curve and the x-axis) is a useful quantity – for example, the area under a velocity–time graph is the distance travelled.



2 Approximating with rectangles

You can estimate that area by covering it with thin **rectangles** and adding their areas – a **Riemann sum** 黎曼和.



3 More rectangles, better estimate

The thinner the rectangles (the more you use), the closer the total gets to the true area.

Worked example. To estimate the area under $y = x$ from 0 to 4 with rectangles: the exact area is a triangle, $\frac{1}{2} \times 4 \times 4 = 8$, and more rectangles get closer and closer to this.

■ 4 The exact area

Making the rectangles infinitely thin gives the **exact** area – this exact value is the **integral**.

Watch out: a Riemann sum with a few rectangles is only an **estimate**. The exact area (the integral) is the limit as the rectangles become infinitely thin and infinitely many.

Practice

Try these in order. The methods are all above.

2.1 State what "the area under a curve" means. [2]

2.2 State how a Riemann sum estimates that area. [2]

2.3 State what happens to the estimate as you use more, thinner rectangles. [1]

2.4 Find the exact area under $y = x$ from 0 to 6 (it is a triangle). [3]

2.5 State what the area under a velocity–time graph represents. [1]

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

(c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

(d) A line forms whenever $A(t) \ge 400$. The number of vehicles in line at time t, for $a \le t \le 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \le t \le 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

6 Integration and the power rule – Part 2

Name: ______ Class: ______ Date: ______

Recall

In Part A you saw that the integral is the exact area under a curve. Now compute it:

- Integration is the **reverse** of differentiation.
- There is a simple rule for powers of x, just like the power rule for derivatives.

Each idea has a worked example before the Practice.

New ideas

1 Integration undoes differentiation

Integration finds a function whose derivative is the one you started with – it is the reverse of differentiating. This reverse is written with the sign $\int$.

2 The power rule for integration

To integrate a power of x, **raise the power by one and divide by the new power**:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

Worked example. $\int x^2 \, dx = \frac{x^3}{3} + C$ (check: differentiating $\frac{x^3}{3}$ gives x^2).

3 The "+ C"

Because many functions share the same derivative (they differ by a constant), we add a constant C when integrating.

4 Definite integral gives area

A **definite integral** (with limits) gives the exact area under the curve between two x-values.

Worked example. The area under $y = x$ from 0 to 2 is $\left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2$.

Watch out: integration **raises** the power and **divides** – the exact opposite of differentiation, which lowers the power and multiplies. Mixing them up is a common slip.

Practice

Try these in order. The methods are all above.

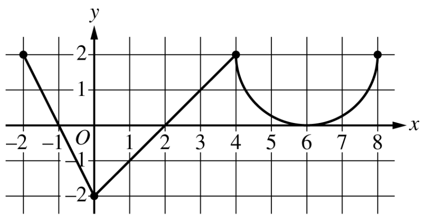
2.1 State how integration relates to differentiation. [1]

2.2 Find $\int x^3\, dx$. [2]

2.3 Find $\int x^4\, dx$. [2]

2.4 State why we add "$+C$" when integrating. [2]

2.5 Find the area under $y = x$ from 0 to 4 using $\left[\frac{x^2}{2}\right]_0^4$. [3]



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

   (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

   (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

   (c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

   (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

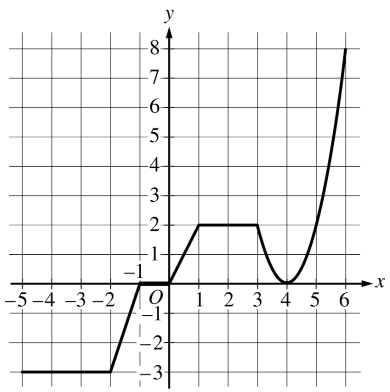
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

7 Differential equations and slope fields – Part 1

Name: ____________ Class: ________ Date: __________

Recall

You have solved ordinary equations to find a number:

- An equation like $2x + 1 = 7$ has the solution $x = 3$.
- But some equations describe how a quantity **changes** – they involve a derivative.

This sheet introduces differential equations. Each idea has a worked example.

New ideas

■ 1 What a differential equation is

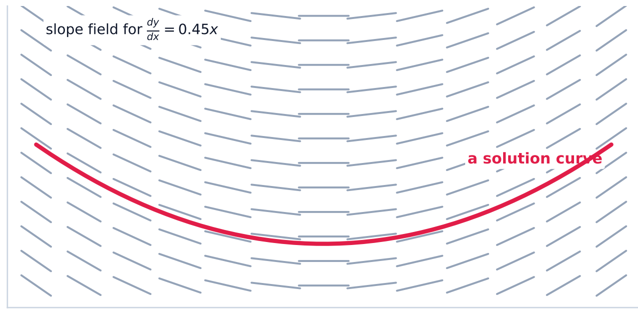
A **differential equation** involves a **derivative** – it tells you the **rate of change**, not the value directly. For example $\dfrac{dy}{dx} = 2x$ says "the slope at each point is $2x$".

■ 2 Its solution is a function

Solving it means finding the **function** y whose derivative matches. For $\dfrac{dy}{dx} = 2x$, one solution is $y = x^2$ (its derivative is $2x$).

■ 3 Slope fields

A **slope field** draws a tiny line at many points showing the slope there. It lets you see the shape of the solutions without solving.



each segment is the local slope $\frac{dy}{dx}$; a solution curve is tangent to every segment it meets

■ 4 A family of solutions

There are usually **many** solutions differing by a constant (like $y = x^2$, $y = x^2 + 1$, ...) – a whole family that follows the slope field.

Watch out: a differential equation gives the **slope** (dy/dx), not the function itself. You must integrate (or otherwise solve) to recover the actual function y.

Practice

Try these in order. The methods are all above.

2.1 State what makes an equation a "differential equation". [2]

2.2 State what it means to solve a differential equation. [2]

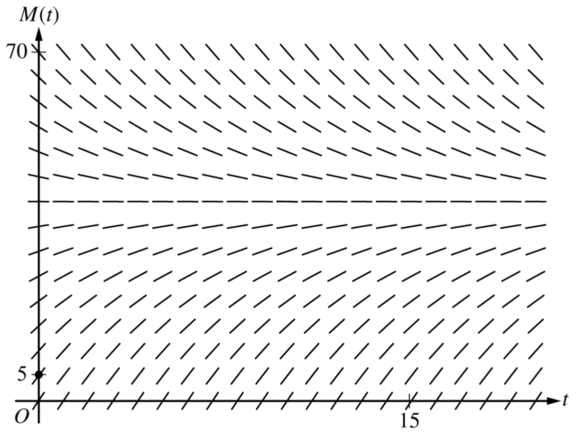
2.3 For $\frac{dy}{dx} = 2x$, verify that $y = x^2$ is a solution. [2]

2.4 State what a slope field shows. [2]

2.5 State why a differential equation usually has many solutions. [1]

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

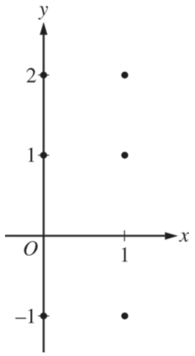
(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



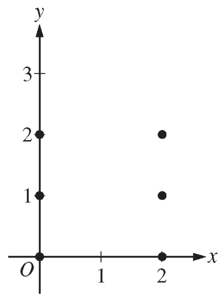
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x - 1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

7 Solving simple differential equations – Part 2

Name: ______ Class: ______ Date: ______

Recall

In Part A you met differential equations. Now solve simple ones:

- Solving means finding the function from its rate of change.
- One special kind describes things that grow by a fixed percentage – like money or populations.

Each idea has a worked example before the Practice.

New ideas

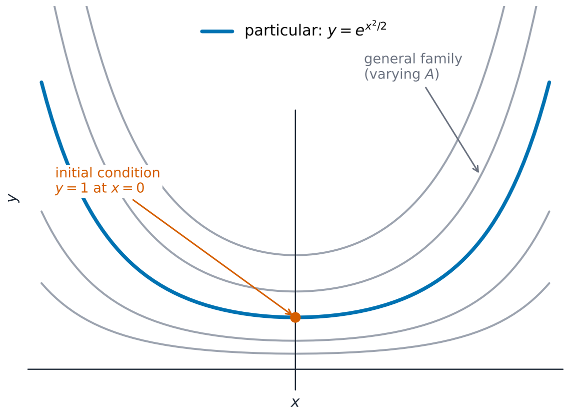
■ 1 Solving by integrating

If $\dfrac{dy}{dx}$ is given, integrate to find y.

Worked example. $\dfrac{dy}{dx} = 3x^2$ integrates to $y = x^3 + C$.

■ 2 A family of curves

The constant C gives a **family** of solution curves; one extra fact (a known point) picks out the right one.



■ 3 Exponential growth

A very common equation is $\dfrac{dy}{dt} = ky$ – "the rate of growth is proportional to how much there is". Its solution is exponential growth, $y = Ce^{kt}$.

■ 4 Where it appears

This models populations, money earning interest, and radioactive decay (with a negative k) – anything that grows or shrinks by a fixed proportion.

Watch out: exponential growth ($\dfrac{dy}{dt} = ky$) means the quantity grows **faster and faster** as it gets bigger, because the rate depends on the current amount. It is not steady, straight-line growth.

Practice

Try these in order. The methods are all above.

2.1 State how you solve $\dfrac{dy}{dx} = f(x)$ for y. [1]

2.2 Solve $\dfrac{dy}{dx} = 2x$. [2]

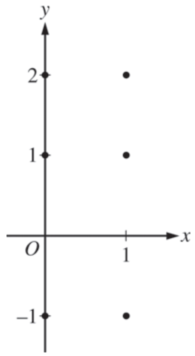
2.3 Solve $\dfrac{dy}{dx} = 4x^3$. [2]

2.4 State the type of growth described by $\dfrac{dy}{dt} = ky$. [1]

2.5 Give two real situations modelled by exponential growth or decay. [2]

4. Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



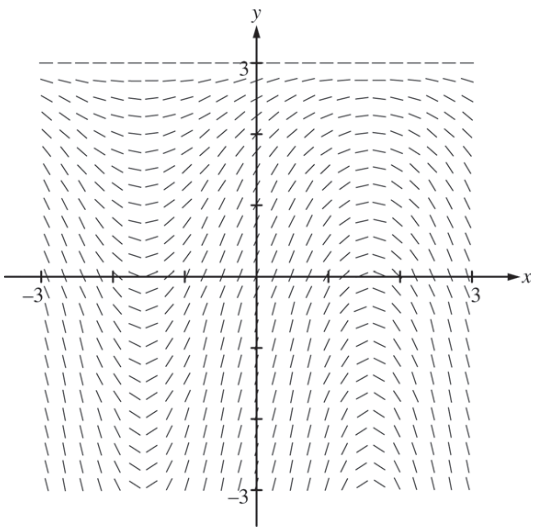
   (b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

   (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

   (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

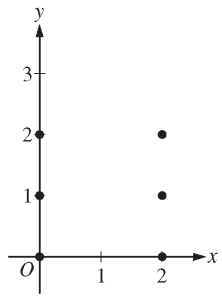
(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

STOP

END OF EXAM

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x - 1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

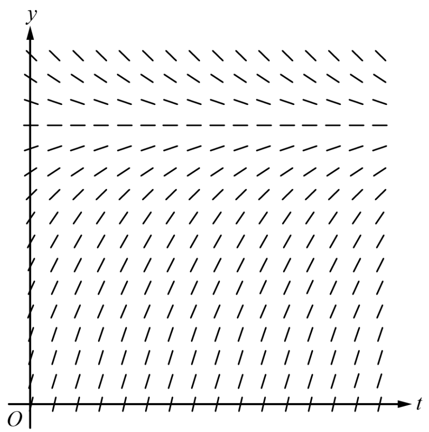


(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

8 Area between two curves – Part 1

Name: ____________ Class: ________ Date: __________

Recall

You have found the area under a single curve. Now the area **between** two curves:

- Two curves can enclose a region between them.
- Finding that area is a common use of integration.

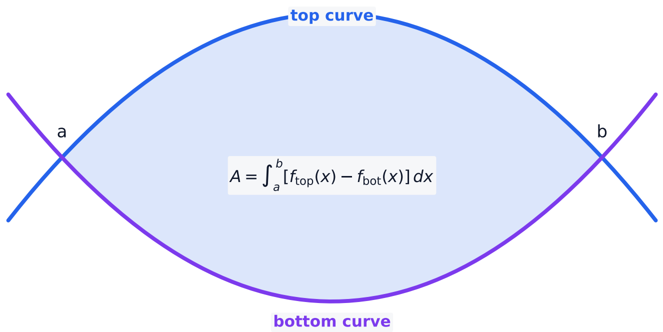
Each idea has a worked example before the Practice.

New ideas

■ 1 Area between two curves

The area between an upper curve and a lower curve is the integral of the **difference**:

$$\text{area} = \int (\text{top} - \text{bottom})\, dx.$$



■ 2 Why subtract

The area under the top curve minus the area under the bottom curve leaves exactly the region between them.

■ 3 A worked example

Worked example. Between $y = 4$ (top) and $y = x^2$ (bottom), where they meet at $x = -2$ and $x = 2$, the area is $\int_{-2}^{2} (4 - x^2)\, dx$ – the constant 4 line minus the parabola.

■ 4 Getting the limits

The two x-values where the curves **cross** are the limits of the integral – set the two functions equal to find them.

Watch out: always subtract **top minus bottom** (the higher curve minus the lower). Subtracting the wrong way round gives a **negative** area, which is a sign you have them reversed.

Practice

Try these in order. The methods are all above.

2.1 Write the formula for the area between an upper and a lower curve. [2]

2.2 Explain why you subtract the two curves. [2]

2.3 For the region between $y = 6$ and $y = x^2$, state what you would integrate. [2]

2.4 State how you find the limits of the integral. [2]

2.5 State what a negative area tells you about your set-up. [1]

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

5. Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

A. Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

AP Calculus AB: **2014**

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

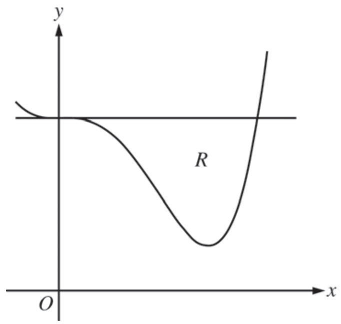
t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

END OF PART A OF SECTION II

8 Volumes and average value – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you found areas with integration. Integration does even more:

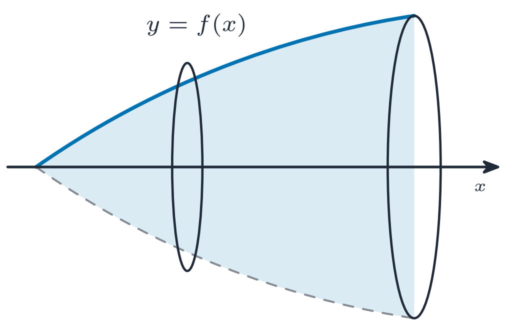
- Spin a flat region around a line and it sweeps out a 3-D solid.
- Integration can find that solid's volume, and the average value of a function.

Each idea has a worked example before the Practice.

New ideas

1 Volumes of revolution

If you spin a region around an axis, it sweeps out a **solid of revolution** – like spinning a rectangle to make a cylinder, or a curve to make a vase shape.



rotate the region around the x-axis to sweep a solid

2 Slicing into discs

Integration finds the volume by adding up many thin **discs**, each a circle of area πr^2 times a tiny thickness.

3 Average value

The **average value** of a function over an interval is its integral divided by the width of the interval – the "typical" height of the curve.

Worked example. The average value of $y = x$ from 0 to 4 is $\frac{1}{4}\int_0^4 x\,dx = \frac{1}{4} \times 8 = 2$.

4 Integration as "adding up"

The common thread: integration **adds up infinitely many tiny pieces** – tiny areas, tiny discs, or tiny contributions to an average.

Watch out: integration is best understood as "adding up many tiny pieces" – tiny rectangles for area, tiny discs for volume. Whenever a quantity is a sum of infinitely many small parts, an integral finds it.

Practice

Try these in order. The methods are all above.

2.1 State what a solid of revolution is. [2]

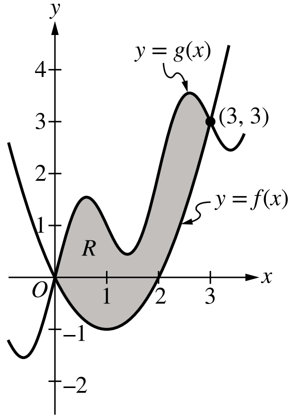
2.2 State the shape you get by spinning a rectangle around one of its sides. [1]

2.3 State how integration finds the volume of a solid of revolution. [2]

2.4 Find the average value of $y = x$ from 0 to 6 (using $\frac{1}{6}\int_0^6 x\,dx$ with $\int_0^6 x\,dx = 18$). [3]

2.5 In one sentence, state what integration does in all these cases. [1]

2. The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

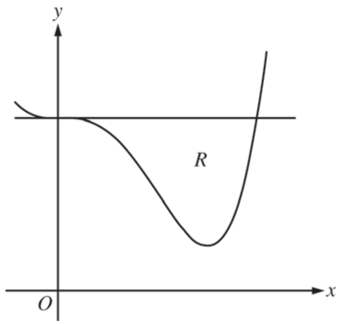
A. Find the area of R. Show the setup for your calculations.

B. Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.

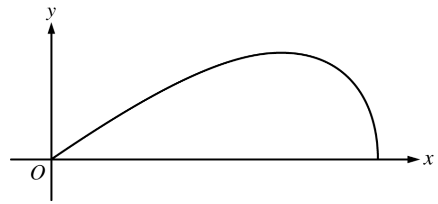
D. It can be shown that $g'(x) = 1 + \pi \cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.

END OF PART A



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4,$ as shown in the figure above.
   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2.$
   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.
   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

END OF PART A OF SECTION II



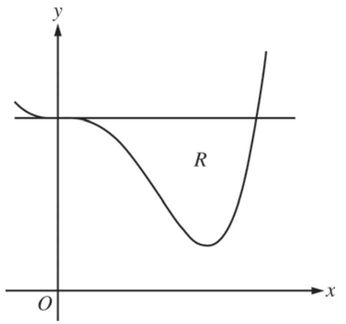
A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\dfrac{dy}{dx} = \dfrac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



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END OF PART A OF SECTION II

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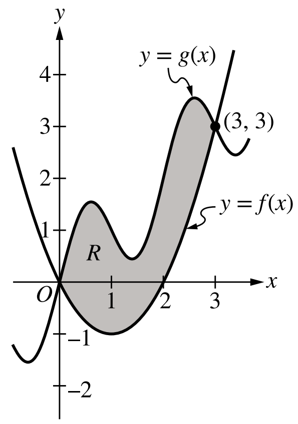

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END OF PART A