Essential skills in a multi-step practical model

Original skill preparation; versioned source scope

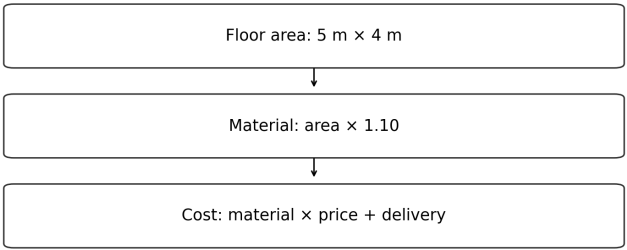
Essential Skills in a Multi-Step Practical Model

- Turn a real situation into a model, stating each **model assumption** 模型假设.
- Use area, percentages, a **unit rate** 单位率 and unit conversions in sequence.
- Read a **scale drawing** 比例图.
- Check a model against data using a **residual** 残差, and make a quick **estimate** 估算 to test reasonableness.

Planning the model

- List the quantities you know, with units, and the one you need.
- Write the steps in order before calculating: area, then material, then cost.
- State assumptions, such as "wastage is 10% of the area" or "the price per unit is constant".

Figure: Planning the model



- Worked steps: Floor area: 5 m $\times$ 4 m $\rightarrow$ Material: area $\times$ 1.10 $\rightarrow$ Cost: material $\times$ price $+$ delivery
- Match each part of the diagram to the text.

Rates, percentages and units

- A percentage allowance multiplies: 10% extra means $\times 1.10$, applied to the quantity named (area, not each side).
- Cost = unit price $\times$ quantity + fixed charges.
- Convert units before combining quantities; write units at each step.

Scale drawings

- A scale of $1 : 50$ means 1 cm on the plan is 50 cm in reality.
- Lengths divide by the scale; areas divide by the scale squared.

Checking and revising a model

- Compare predictions with observations: **residual** = observed − predicted.
- A constant residual suggests the fixed part (intercept) is wrong; a residual that grows with the input suggests the rate (slope) is wrong.
- A rough **estimate** in round numbers catches errors in the calculation.

Worked example: flooring a room

- Area: $5 \times 4 = 20$ m^2. Material: $1.10 \times 20 = 22$ m^2.
- Cost: $35 \times 22 + 60 = 770 + 60 = 830$ yuan.
- **Estimate**: about $20 \times 35 = 700$, plus extras: 830 is reasonable.
- On a $1 : 50$ plan: $500 \div 50 = 10$ cm by $400 \div 50 = 8$ cm.

Worked example: revising a delivery model

- Every **residual** is $+5$, the same at each distance.
- So raise the fixed part: $C = 11 + 4d$.
- If instead the residuals grew with distance (say $+2$, $+4$, $+6$), the per-kilometre rate would be too low. Either way, check the actual fee rules before trusting the revision.

Common errors

- Applying a 10% allowance to each side length instead of the area.
- Forgetting a fixed charge.
- Dividing an area by the scale instead of the scale squared.

Common errors (2)

- Changing the slope when the residuals show a constant gap.
- Skipping the reasonableness check.

Key terms

- **model assumption**: A condition a model takes to be true, such as a constant price.
- **unit rate**: The amount for one unit, such as yuan per square metre.
- **scale drawing**: A drawing in which every length is reduced by the same factor.
- **residual**: Observed value minus the model's prediction.

Check yourself

- A wall is 2 m by 3 m, and 1 L of paint covers 10 m^2. How much paint is needed for one coat?
- On a $1:200$ plan, a wall is 3 cm long. How long is it really?
- A model's residuals are -1, -3, -5 as the input rises. Which part of the model should you check?