Sampling inference, expected value and counting

Original skill preparation; versioned source scope

Sampling Inference, Expected Value and Counting

- Count arrangements, telling a **permutation** 排列 from a **combination** 组合.
- Find probabilities, including **conditional probability** 条件概率 and draws without replacement.
- Calculate an **expected value** 期望值.
- Interpret a survey's **margin of error** 误差范围 and what a study can claim.

Counting

- Multiplication principle: if one choice has m options and the next n, together there are mn.
- Order matters (a **permutation**): ${}_nP_r = n(n-1)\cdots(n-r+1)$.
- Order does not matter (a **combination**): ${}_nC_r = \frac{{}_nP_r}{r!}$, since each group was counted $r!$ times.

Figure: Counting

Choose 2 of 4: six unordered pairs; AB and BA count once

AB	AC	AD
BC	BD	CD

- Choosing 2 of 4 gives six pairs, because AB and BA are the same pair
- Match each part of the diagram to the text.

Probability without replacement

- Without replacement, the second draw depends on the first: the totals change.
- $P(A \text{ and } B) = P(A) \times P(B \mid A)$.
- "Exactly one" can happen in more than one order: add the cases.

Expected value

- Multiply each outcome by its probability and add.
- Net gain = expected payout − cost.
- The expected value is a long-run average; a single play may never give that amount.

$$E(X) = \sum x\,P(x)$$

Samples and studies

- An estimate with a **margin of error** gives an interval of plausible values for the population that was sampled.
- A large sample of volunteers can still be biased; random selection reduces bias.
- Random assignment to treatments supports a causal claim; random sampling supports generalising.

Worked example: roles, committees and draws

- Six students choose a chair and a different secretary: $6 \times 5 = 30$ ways (order matters).
- An unranked pair of two: $\dfrac{30}{2} = 15$, since each pair was counted twice.
- A bag has 3 red and 2 blue. $P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$.
- $P(\text{exactly one red}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{3}{5}$.

Worked example: a game and a survey

- $E(\text{payout}) = 12(0.1) + 3(0.4) + 0(0.5) = 2.4$ tokens; expected net gain $= 0.4$ token per play.
- The actual net results are 10, 1 or -2: never 0.4.
- A random sample of 500 library members finds 62% support, margin 4 points: the interval is 58% to 66% of members, not of all town residents.

Common errors

- Using a **combination** when order matters, or the reverse.
- Keeping the same denominator for a second draw without replacement.
- Counting only one order for "exactly one".

Common errors (2)

- Expecting the expected value on a single play.
- Generalising a survey beyond the sampled population.

Key terms

- **permutation**: An arrangement in which order matters.
- **combination**: A selection in which order does not matter.
- **conditional probability**: The probability of an event given that another has happened.
- **expected value**: The long-run average outcome, $\sum x\,P(x)$.

Check yourself

- In how many orders can 4 books stand on a shelf?
- How many ways can you choose 2 of 5 people (unordered)?
- What is the expected value of one roll of a fair six-sided die?