Conics, circle relationships and geometric measurement

Original skill preparation; versioned source scope

Conics, Circle Relationships and Geometric Measurement

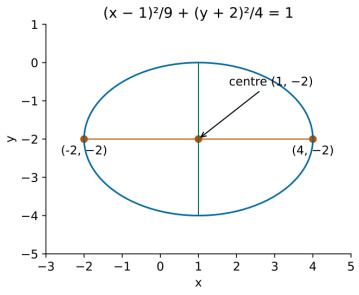
- Read the centre and **semiaxes** 半轴 of an **ellipse** 椭圆.
- Recognise a **hyperbola** 双曲线 and find the vertex and **focus** 焦点 of a **parabola** 抛物线.
- Use lengths of a **tangent** 切线 and a **secant** 割线 drawn from one point.
- Find volumes and surface areas, and how they scale.

Circles and ellipses

- Centre (h, k); semiaxis a across, b up and down. If $a = b$, it is a circle of radius a.
- The ends of the axes are $(h \pm a, k)$ and $(h, k \pm b)$.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Figure: Circles and ellipses



- The denominators 9 and 4 give semi-axes 3 across and 2 up and down
- Match each part of the diagram to the text.

Parabolas and hyperbolas

- $y^2 = 4px$: vertex at the origin, **focus** $(p, 0)$; it opens right if $p > 0$, left if $p < 0$. Likewise $x^2 = 4py$ has focus $(0, p)$.
- A minus sign between the squared terms, as in $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, gives a **hyperbola**.
- Classify by the signs: both squared terms positive (**ellipse** or circle), opposite signs (hyperbola), only one squared term (parabola).

Tangents, secants and chords

- From an outside point, a **tangent** of length t and a secant with outside part e and whole length w satisfy $t^2 = e\,w$.
- Two secants from the same point: $e_1 w_1 = e_2 w_2$.
- Two chords crossing inside a circle: the products of their parts are equal.
- A tangent is perpendicular to the radius at the point of contact.

Measurement and scaling

- Cylinder: $V = \pi r^2 h$, surface area $2\pi r^2 + 2\pi rh$. Cone: $V = \frac{1}{3}\pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$, area $4\pi r^2$.
- If every length is multiplied by k, areas are multiplied by k^2 and volumes by k^3.

Worked example: reading equations

- $\dfrac{(x+2)^2}{25} + \dfrac{(y-1)^2}{9} = 1$: centre $(-2, 1)$, **semiaxes** 5 and 3.
- Axis ends: $(-7, 1)$, $(3, 1)$, $(-2, -2)$, $(-2, 4)$.
- $y^2 = -12x$: $4p = -12$, so $p = -3$. Vertex $(0, 0)$, **focus** $(-3, 0)$; it opens to the left.

Worked example: a secant and a cylinder

- A **tangent** from a point has length 6 cm; a secant from it has outside part 4 cm. Then $w = \dfrac{6^2}{4} = 9$ cm, and the part inside the circle is $9 - 4 = 5$ cm.
- A cylinder with $r = 3$ cm and $h = 8$ cm: $V = \pi(9)(8) = 72\pi$ cm^3; area $= 18\pi + 48\pi = 66\pi$ cm^2.
- Doubling every length: volume $\times 8 = 576\pi$ cm^3; area $\times 4 = 264\pi$ cm^2.

Common errors

- Reading a^2 as the semiaxis instead of a.
- Getting the sign of the centre wrong.
- Using the inside part of a secant instead of its whole length.

Common errors (2)

- Mixing up p and $4p$ for a **parabola**.
- Multiplying volume by k instead of k^3.

Key terms

- **ellipse**: An oval curve with equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.
- **semiaxes**: The distances a and b from the centre to the ends of an **ellipse**'s axes.
- **hyperbola**: A conic with two separate branches, with a minus sign between the squared terms.
- **parabola**: A conic with only one squared variable.

Check yourself

- Classify $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and $x^2 + y^2 = 16$.
- Find the circumference and area of the circle $x^2 + y^2 = 16$.
- Two secants from one point: the first has outside part 3 and whole length 12; the second has outside part 4. Find its whole length.