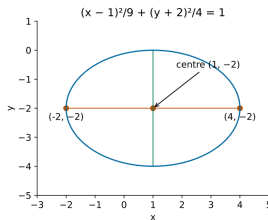


Conics, circle relationships and geometric measurement

Original skill preparation; versioned source scope

Method and conditions



- An ellipse $(x - h)^2/a^2 + (y - k)^2/b^2 = 1$ has centre (h, k) .
- Completing squares; squared semiaxes; circle geometry; volume units.

Worked method 1

Read and reason

An ellipse $(x - h)^2/a^2 + (y - k)^2/b^2 = 1$ has centre (h, k) . Semiaxes are square roots of the positive denominators. A parabola $y^2 = 4px$ has focus $(p, 0)$ and directrix $x = -p$. Distinguish these equations from a hyperbola with a difference of squares.

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

For $(x + 2)^2/25 + (y - 1)^2/9 = 1$, state centre, horizontal and vertical semiaxes, and four axis endpoints. For $y^2 = -12x$, state vertex and focus.

Reasoning 1.1

Read and reason

Centre is $(-2, 1)$, semiaxes 5 and 3, endpoints $(-7, 1)$, $(3, 1)$, $(-2, -2)$, $(-2, 4)$. In the parabola, $4p = -12$, so $p = -3$; vertex $(0, 0)$, focus $(-3, 0)$.

Attempt 2.1

Read and reason

From one exterior point a tangent has length 6 cm. A secant has outside segment 4 cm. Find its whole and inside lengths.

Reasoning 2.1

Read and reason

Use tangent–secant relation $t^2 = ew$. Rearrange $w = t^2/e = (6\text{ cm})^2/(4\text{ cm}) = 9\text{ cm}$. Inside length is $w - e = 9\text{ cm} - 4\text{ cm} = 5\text{ cm}$. The product uses whole length, not just inside.

Attempt 3.1

Read and reason

A cylindrical container has radius 3 cm and height 8 cm. Find its volume and total closed surface area. What happens to each if every length doubles?

Reasoning 3.1

Read and reason

$$V = \pi r^2 h = \pi (3 \text{ cm})^2 (8 \text{ cm}) = 72\pi \text{ cm}^3.$$

$A = 2\pi r^2 + 2\pi rh = 2\pi (3 \text{ cm})^2 + 2\pi (3 \text{ cm})(8 \text{ cm}) = 66\pi \text{ cm}^2$. Doubling all lengths multiplies volume by 8 and area by 4.

Attempt 4.1

Read and reason

Classify $x^2/16 - y^2/9 = 1$ and $x^2 + y^2 = 16$. Find the circle circumference and area.
For two secants from the same exterior point, one has outside/whole lengths 3/12 cm and the other outside length 4 cm: find its whole and inside lengths.

Reasoning 4.1

Read and reason

The difference of squared terms defines a hyperbola centred at the origin. The sum with equal coefficients defines a circle of radius 4. Its circumference is $C = 2\pi r = 8\pi$ and area $A = \pi r^2 = 16\pi$ in coordinate length and square-length units respectively. Secant products match: $e_1 w_1 = e_2 w_2$.

Reasoning 4.2

Read and reason

Thus $w_2 = (3 \text{ cm})(12 \text{ cm})/(4 \text{ cm}) = 9 \text{ cm}$; inside length is $9 - 4 = 5 \text{ cm}$. Outside times inside would apply the wrong relation.

Transfer and limits

Read and reason

Completing squares; squared semiaxes; circle geometry; volume units.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.