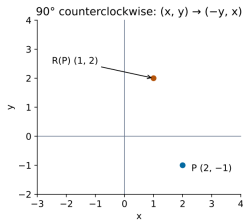


Plane transformations, coordinate reasoning and proof

Original skill preparation; versioned source scope

Method and conditions



- Rigid motions preserve lengths and angles.
- Coordinate differences; triangle congruence; angles on parallel lines.

Worked method 1

Read and reason

Rigid motions preserve lengths and angles. A 90-degree anticlockwise rotation sends (x, y) to $(-y, x)$ about the origin. Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Equal angles establish similarity, but congruence also fixes size.

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

Rotate $A = (2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$.
Reverse the order and compare endpoints.

Reasoning 1.1

Read and reason

Rotate first to $(3, 2)$, then translate to $(7, 3)$. Translate first to $(6, -2)$, then rotate to $(2, 6)$. The endpoints differ, so transformation order matters.

Attempt 2.1

Read and reason

$A = (-1, 2)$ and $B = (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation.

Reasoning 2.1

Read and reason

$d = \sqrt{(5 + 1)^2 + (10 - 2)^2} = 10$. Midpoint $M = ((-1 + 5)/2, (2 + 10)/2) = (2, 6)$. Segment slope is $8/6 = 4/3$, so perpendicular slope is $-3/4$. The bisector is $y - 6 = -(3/4)(x - 2)$.

Attempt 3.1

Read and reason

Two triangles have angles $40^\circ, 60^\circ, 80^\circ$. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas.

Reasoning 3.1

Read and reason

The triangles are similar by equal angles, not congruent because corresponding longest sides differ. Length ratio is $15/9 = 5/3$ and larger-to-smaller area ratio is $25/9$.

Attempt 4.1

Read and reason

Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3,4,5 cm.

Reasoning 4.1

Read and reason

Use the same compass radius, greater than half AB , to draw arcs centred at A and B meeting at P and Q on opposite sides. Draw PQ . $PA=PB$ and $QA=QB$ by equal radii. Thus both P and Q lie on the locus equidistant from A and B ; PQ is the perpendicular bisector.

Reasoning 4.2

Read and reason

Equivalently use congruent triangles and the common chord to establish perpendicularity and bisection. A ruler reading on one picture does not prove the general locus. SSS proves the two stated triangles congruent, fixing size as well as shape; Pythagoras also shows these particular triangles are right-angled.

Attempt 5.1

Read and reason

Two parallel lines are cut by a transversal. One corresponding angle is 68° . Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step.

Reasoning 5.1

Read and reason

Parallelism gives the matching corresponding angle 68° . The adjacent pair forms a straight angle, so the other is $180^\circ - 68^\circ = 112^\circ$. Equal corresponding angles require the stated parallel lines; the straight-angle sum uses adjacency on one line. A similar-looking sketch without parallelism would not justify the first equality.

Transfer and limits

Read and reason

Coordinate differences; triangle congruence; angles on parallel lines.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.