Trigonometric graphs, identities and general triangles

Original skill preparation; versioned source scope

Trigonometric Graphs, Identities and General Triangles

- Use sine, cosine and tangent in right triangles and on the unit circle.
- Read the **amplitude** 振幅, **period** 周期 and **midline** 中线 of a sine or cosine graph.
- Use the **sine rule** 正弦定理 and the **cosine rule** 余弦定理 in any triangle.
- Use the **included angle** 夹角 to find a triangle's area, and basic identities.

Right triangles and the unit circle

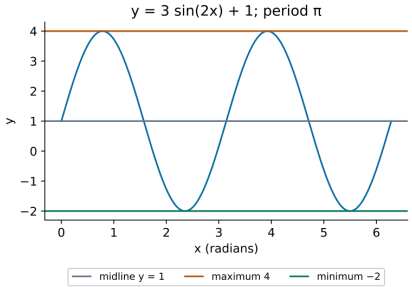
- $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$.
- On the unit circle, the point at angle θ is $(\cos\theta, \sin\theta)$, so $\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.
- Sine is positive in quadrants I and II; cosine in I and IV. So $\sin\theta = \frac{1}{2}$ has two answers in a full turn: 30° and 150°.

Graphs of sine and cosine

- Amplitude $|a|$: half the distance from maximum to minimum. A negative a reflects the wave; the amplitude is still positive.
- Period $\dfrac{2\pi}{b}$ (in radians) or $\dfrac{360^\circ}{b}$.
- Midline $y = d$; range from $d - |a|$ to $d + |a|$.

$$y = a\sin(bx) + d$$

Figure: Graphs of sine and cosine



- Amplitude 3 about the midline y = 1; the factor 2 makes the period π
- Read the axes and units first.

Triangles that are not right-angled

- Use the sine rule when you know a side and its opposite angle.
- Use the **cosine rule** with two sides and the angle between them, or with three sides.
- The area formula needs the angle between the two given sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad c^2 = a^2 + b^2 - 2ab\cos C \qquad \text{Area} = \tfrac{1}{2}ab\sin C$$

Worked example: a graph

- Amplitude 4 (the minus sign reflects the wave); period $\dfrac{2\pi}{2} = \pi$; midline $y = 3$.
- Range: $3 - 4 = -1$ to $3 + 4 = 7$.
- At $x = \frac{\pi}{4}$: $y = -4\sin\frac{\pi}{2} + 3 = -4 + 3 = -1$, a minimum.

Worked example: two general triangles

- $c^2 = 36 + 100 - 2(6)(10)\cos 60^\circ = 136 - 60 = 76$, so $c = 2\sqrt{19} \approx 8.72$ cm.
- Area $= \frac{1}{2}(6)(10)\sin 60^\circ = 30 \cdot \frac{\sqrt{3}}{2} = 15\sqrt{3} \approx 26.0$ cm^2.
- Another triangle has 8 cm opposite 30° and an angle of 45°. Sine rule: $b = \dfrac{8\sin 45^\circ}{\sin 30^\circ} = \dfrac{8(\sqrt{2}/2)}{1/2} = 8\sqrt{2}$ cm.

Common errors

- Using SOH-CAH-TOA in a triangle that is not right-angled.
- Giving only the acute solution of $\sin\theta = \frac{1}{2}$.
- Writing a negative amplitude.

Common errors (2)

- Using the period $2\pi b$ instead of $\dfrac{2\pi}{b}$.
- Using a side angle in the area formula instead of the **included angle**.

Key terms

- **amplitude**: Half the distance between the maximum and minimum of a wave.
- **period**: The length of one full cycle.
- **midline**: The horizontal line halfway between maximum and minimum.
- **sine rule**: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

Check yourself

- Find every θ with $0° \leq \theta < 360°$ and $\sin\theta = \frac{1}{2}$.
- What is the period of $y = \cos(3x)$, in radians?
- A right triangle has legs 9 and 12. Find $\sin$ of the angle opposite 9.