Sequences, finite sums, exponentials and logarithms

Original skill preparation; versioned source scope

Sequences, Finite Sums, Exponentials and Logarithms

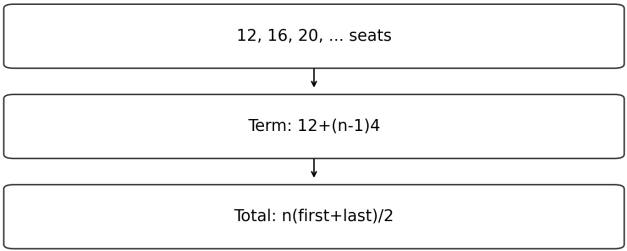
- Find terms and sums of an **arithmetic sequence** 等差数列.
- Find terms and sums of a **geometric sequence** 等比数列 with a **common ratio** 公比.
- Write exponential growth and decay models.
- Use a **logarithm** 对数, and keep its **logarithm argument** 真数 positive.

Arithmetic sequences

- Each term adds the same difference d.
- The sum is the number of terms times the average of the first and last.

$$a_n = a_1 + (n-1)d \qquad S_n = \frac{n(a_1 + a_n)}{2}$$

Figure: Arithmetic sequences



- Worked steps: 12, 16, 20, ... seats → Term: 12+(n-1)4 → Total: n(first+last)/2
- Match each part of the diagram to the text.

Geometric sequences

- Each term multiplies by the same ratio r.
- If $r = 1$, all terms are equal and $S_n = na_1$.

$$a_n = a_1 r^{n-1} \qquad S_n = \frac{a_1(1 - r^n)}{1 - r}, \ r \neq 1$$

Exponential models

- $N(t) = N_0 b^{t/T}$: start N_0, factor b for each period T.
- Growth of $r\%$ per period: $b = 1 + r/100$; decay: $b = 1 - r/100$.

Logarithms

- $\log_b y = x$ means $b^x = y$. The argument y must be positive.
- $\log(MN) = \log M + \log N$; $\log(M/N) = \log M - \log N$; $\log(M^k) = k \log M$.
- There is no rule for $\log(M + N)$.
- To solve $b^x = c$, take logs: $x = \dfrac{\log c}{\log b}$.

Worked example: seats in rows

- $d = 4$, so row 8 has $a_8 = 12 + 7(4) = 40$ seats.
- Total in 8 rows: $S_8 = \dfrac{8(12 + 40)}{2} = 208$ seats.
- This assumes every row adds exactly 4 seats.

Worked example: geometric terms and logarithms

- For 5, 15, 45, …: $a_5 = 5(3)^4 = 405$, and $S_5 = \dfrac{5(1-3^5)}{1-3} = \dfrac{5(-242)}{-2} = 605$.
- $\log_2(x-3) = 5$: need $x > 3$; then $x - 3 = 2^5 = 32$, so $x = 35$.
- $3^x = 20$: $x = \dfrac{\log 20}{\log 3} \approx \dfrac{1.301}{0.477} \approx 2.73$.

Common errors

- Using n instead of $n-1$ in the term formula.
- Using the geometric sum formula when $r=1$.
- Splitting $\log(x+3)$ into $\log x + \log 3$.

Common errors (2)

- Forgetting that the logarithm's argument must be positive.
- Mixing up the difference and the ratio.

Key terms

- **arithmetic sequence**: A sequence with a constant difference between terms.
- **geometric sequence**: A sequence with a constant ratio between terms.
- **common ratio**: The number each term of a **geometric sequence** is multiplied by.
- **logarithm**: The power to which a base must be raised to give a number.

Check yourself

- Find the 7th term of 3, 7, 11, ….
- Evaluate $\log_3 81$.
- Find $2 + 4 + 8 + \cdots + 64$.