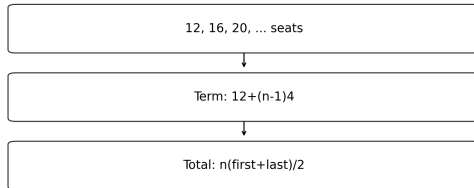


Sequences, finite sums, exponentials and logarithms

Original skill preparation; versioned source scope

Method and conditions



- Arithmetic sequence : $a_n = a_1 + (n-1)d$.
- Index notation; exponent rules; finite sequences.

Worked method 1

Read and reason

Arithmetic sequence : $a_n = a_1 + (n - 1)d$. Its sum is $S_n = n(a_1 + a_n)/2$.

Geometric sequence : $a_n = a_1 r^{n-1}$ and $S_n = a_1(1 - r^n)/(1 - r)$ for $r \neq 1$. A

logarithm reverses exponentiation: $\log_b a = c$ means $b^c = a$, with $a > 0$, $b > 0$, $b \neq 1$.

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

Seats in successive rows number 12, 16, 20, Find the eighth row and the total in the first eight rows. State the model assumption.

Reasoning 1.1

Read and reason

Constant difference is 4 seats per row. $a_8 = a_1 + 7d = 12 + 7(4) = 40$ seats.

$S_8 = 8(a_1 + a_8)/2 = 8(12 + 40)/2 = 208$ seats. This assumes the same four-seat increment through all eight rows.

Attempt 2.1

Read and reason

A geometric sequence starts 5, 15, 45, Find its fifth term and sum of the first five terms. Explain what changes if the ratio is 1.

Reasoning 2.1

Read and reason

$a_5 = a_1 r^4 = 5(3)^4 = 405$. $S_5 = a_1(1 - r^5)/(1 - r) = 5(1 - 243)/(1 - 3) = 605$. When $r = 1$, the fraction has zero denominator and the correct sum is na_1 , here 25 for five identical terms.

Attempt 3.1

Read and reason

Solve $\log_2(x - 3) = 5$ and explain why $\log_2(x + 3)$ cannot generally be split into $\log_2 x + \log_2 3$.

Reasoning 3.1

Read and reason

The domain requires $x > 3$. Exponential form gives $x - 3 = 32$, so $x = 35$ and the argument is positive. The sum of logs corresponds to the product $3x$, not $x + 3$; at $x = 1$, the proposed equality would equate 2 with $\log_2 3$.

Attempt 4.1

Read and reason

A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t , find $N(4)$, and distinguish factor from percentage growth.

Reasoning 4.1

Read and reason

$N(t) = 120(1.5)^{t/2}$ cells. $N(4) = 120(1.5)^2 = 270$ cells. The growth is 50% per two hours, not 150%; the four-hour factor is 2.25. The hourly factor would be $\sqrt{1.5}$.

Transfer and limits

Read and reason

Index notation; exponent rules; finite sequences.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.