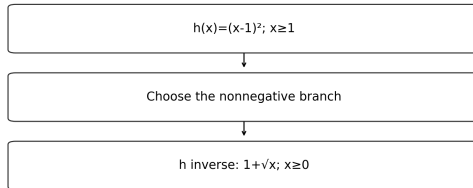


# Functions, composition, transformations and inverses

Original skill preparation; versioned source scope

## Method and conditions



- Composition                      applies the inside rule first.
- Function notation; substitution; equation solving.

## Worked method 1

### Read and reason

Composition applies the inside rule first. If  $f(u) = 1/(u - 2)$  and  $g(x) = x^2 + 2$ ,  $f(g(x)) = 1/x^2$ , with  $x \neq 0$ . An inverse reverses input and output on a one-to-one domain. Restrict  $h(x) = (x - 1)^2$  to  $x \geq 1$  before taking  $h^{-1}(x) = 1 + \sqrt{x}$ .

## Original context 1

Read and reason

## Task context

### Read and reason

Each task below states its own complete data. Values may differ from the worked example.

## Attempt 1.1

### Read and reason

Let  $f(u) = 1/(u - 3)$  and  $g(x) = x^2 + 3$ . Find  $f(g(x))$  with domain. Find  $g(f(4))$  and explain why these compositions differ.

## Reasoning 1.1

### Read and reason

$f(g(x)) = 1/(x^2 + 3 - 3) = 1/x^2$ , excluding  $x = 0$ . For the other order,  $f(4) = 1$  and  $g(1) = 4$ . Composition order changes both operation and possibly domain;  $f(g(4)) = 1/16$  is different.

## Attempt 2.1

### Read and reason

Restrict  $h(x) = (x + 2)^2$  to  $x \leq -2$ . Find its inverse, including domain and range. Explain why the positive-root branch is wrong.

## Reasoning 2.1

### Read and reason

Solve  $y = (x + 2)^2$  with  $x + 2 \leq 0$  to get  $x = -2 - \sqrt{y}$ . Thus  $h^{-1}(x) = -2 - \sqrt{x}$ , domain  $x \geq 0$ , range  $y \leq -2$ . The plus branch returns values outside the chosen original domain.

## Attempt 3.1

### Read and reason

The graph of  $f(x) = x^2$  is changed to  $g(x) = 3f(x + 4) - 2$ . Find its vertex and range, checking one mapped point.

## Reasoning 3.1

### Read and reason

$g(x) = 3(x + 4)^2 - 2$  has vertex  $(-4, -2)$  and range  $y \geq -2$ . Old point  $(1, 1)$  maps to  $(-3, 1)$  because  $3(1) - 2 = 1$ ; substitution confirms  $g(-3) = 1$ .

# Transfer and limits

## Read and reason

Function notation; substitution; equation solving.

## Check yourself

### Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.