Rational equations, radical branches and systems

Original skill preparation; versioned source scope

Rational Equations, Radical Branches and Systems

- Solve rational equations and state each **restriction 限制条件** on the variable.
- Solve radical equations, checking each **candidate root 候选根** and rejecting an **extraneous solution 增根**.
- Use the **principal square root 算术平方根** correctly.
- Solve systems that include a curve, and describe regions of inequalities.

Rational equations

- Before anything else, list the values that make a denominator zero.
- Multiply every term by the lowest common denominator, then solve.
- Reject any answer on your list. An equation can end with no solution.
- When simplifying a rational expression, the original restrictions remain.

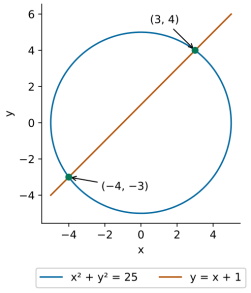
Radical equations

- $\sqrt{x}$ means the principal (non-negative) root, so $\sqrt{A} = x$ requires $x \geq 0$.
- Isolate the root, square both sides, solve, and check every candidate in the original equation.
- $x^2 = 9$ has two branches, $x = 3$ or $x = -3$; but $\sqrt{9}$ is only 3.

Systems with a curve

- Substitute the linear equation into the curved one to get a single equation, often a quadratic.
- Find the second coordinate from the linear equation.
- The number of real solutions of that quadratic is the number of intersection points.

Figure: Systems with a curve



- Substituting the line into the circle gives both crossing points
- Match each part of the diagram to the text.

Systems of inequalities

- Draw each boundary: solid for $\leq$ or $\geq$, dashed for $<$ or $>$.
- The solution is the region satisfying all the inequalities; test a point to check.

Worked example: a circle and a line

- Substitute: $x^2 + (x-1)^2 = 25$, so $2x^2 - 2x - 24 = 0$, that is $x^2 - x - 12 = 0$.
- $(x-4)(x+3) = 0$, so $x = 4$ or $x = -3$.
- Points: $(4, 3)$ and $(-3, -4)$. Check: $16 + 9 = 25$ and $9 + 16 = 25$.

Worked example: rejecting candidates

- $\sqrt{3x+4} = x$: square to get $x^2 - 3x - 4 = (x-4)(x+1) = 0$. Check: $\sqrt{16} = 4$ works; $\sqrt{1} = 1 \neq -1$, so reject -1. Answer: $x = 4$.
- $\dfrac{x}{x-2} = \dfrac{2}{x-2} + 3$, with $x \neq 2$: multiply by $x-2$ to get $x = 2 + 3(x-2)$, so $x = 2$.
- But $x = 2$ is excluded, so this equation has no solution.

Common errors

- Not listing the restrictions before solving.
- Keeping an answer that makes a denominator zero.
- Forgetting to check candidates after squaring.

Common errors (2)

- Writing $\sqrt{9} = \pm 3$.
- Giving only x-values when the question asks for points.

Key terms

- **restriction**: A value the variable cannot take, such as one that makes a denominator zero.
- **candidate root**: A possible solution that must be checked.
- **extraneous solution**: A candidate that comes from the method but does not satisfy the original equation.
- **principal square root**: The non-negative square root, written $\sqrt{x}$.

Check yourself

- Solve $\frac{1}{x} + \frac{1}{2} = 1$.
- Solve $\sqrt{x+6} = x$.
- How many points do $y = x^2$ and $y = -1$ share?