Vectors and matrices as organised quantitative models

Original skill preparation; versioned source scope

Vectors and Matrices as Organised Quantitative Models

- Find a **displacement vector** 位移向量 and its **magnitude** 模.
- Add vectors and multiply them by a **scalar** 标量.
- Use a **matrix** 矩阵 to organise data, and multiply matrices when the sizes allow.
- Calculate a **dot product** 点积.

Vectors

- A vector has size and direction. In components, the displacement from A to B is $B - A$.
- Its magnitude is $\sqrt{x^2 + y^2}$, by Pythagoras.
- Add vectors component by component; a **scalar** multiplies every component.
- The direction matters: $\overrightarrow{BA} = -\overrightarrow{AB}$.

Matrices: size and simple operations

- A matrix with m rows and n columns has size $m \times n$.
- Add or subtract only matrices of the same size, entry by entry.
- A **scalar** multiplies every entry.

Matrix multiplication

- AB is defined only if the number of columns of A equals the number of rows of B. An $m \times n$ times an $n \times p$ gives $m \times p$.
- Each entry is a row of A times a column of B: multiply matching entries and add.
- In general $AB \neq BA$, and one may exist while the other does not.

Figure: Matrix multiplication

Quantity rows × price column (4, 5)

Row	Item A	Item B	Total cost
List 1	2	1	13
List 2	1	3	19

- Quantity rows times the price column give each total; here the prices are 4 and 5
- Match each part of the diagram to the text.

Dot product and determinant

- A **dot product** of zero means the vectors are perpendicular.
- A 2×2 matrix with determinant zero has no inverse.

$$(a, b) \cdot (c, d) = ac + bd \qquad \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Worked example: displacement

- $\overrightarrow{AB} = (7-1,\ 6-(-2)) = (6, 8)$ m.
- Magnitude: $\sqrt{36+64} = 10$ m.
- Adding the displacement $(-3, 2)$ m to B gives the point $(4, 8)$.
- **Dot product** $(6, 8) \cdot (-3, 2) = -18 + 16 = -2$: not zero, so not perpendicular.

Worked example: an order matrix

- The orders cost 20 yuan and 30 yuan. The items must be in the same order in the rows and in the price column.
- If A is 2×3 and B is 3×4, then AB is 2×4, but BA is undefined (4 columns against 2 rows).

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 3(2) + 2(7) \\ 1(2) + 4(7) \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

Common errors

- Subtracting the points in the wrong order.
- Adding magnitudes instead of components.
- Multiplying matrices entry by entry.

Common errors (2)

- Assuming $AB = BA$.
- Mismatching the item order between rows and columns.

Key terms

- **displacement vector**: The change in position from one point to another.
- **magnitude**: The length of a vector.
- **scalar**: An ordinary number that multiplies a vector or matrix.
- **matrix**: A rectangular array of numbers.

Check yourself

- Find the magnitude of $(5, 12)$.
- Find $2(1, -3) + (4, 1)$.
- What is the size of a 3×2 matrix times a 2×5 matrix?