Mathematics: models, functions and higher maths

Original skill preparation; versioned source scope

ACT Mathematics: Models, Functions and Higher Maths

- Calculate with real and complex numbers, using the **complex conjugate** 复共轭 to divide.
- Work with functions, including **composition** 函数复合.
- Solve problems in geometry and trigonometry.

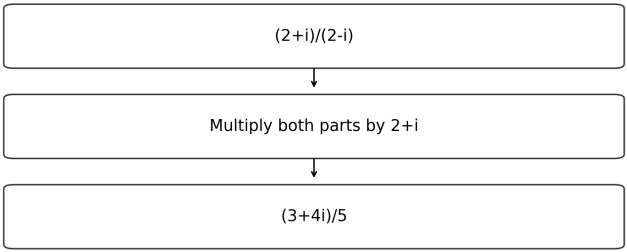
ACT Mathematics: Models, Functions and Higher Maths (2)

- Use probability, including **independent** 独立的 events, and statistics.
- Build and use a **model** 模型 of a real situation.

Number and complex numbers

- $i^2 = -1$. Add and multiply complex numbers like binomials, replacing i^2 with -1.
- To divide, multiply top and bottom by the conjugate of the denominator: the conjugate of $a + bi$ is $a - bi$.
- $(a + bi)(a - bi) = a^2 + b^2$, a real number.
- Rational powers: $x^{m/n} = (\sqrt[n]{x})^m$; negative powers: $x^{-n} = 1/x^n$.

Figure: Number and complex numbers



- Worked steps: (2+i)/(2-i) → Multiply both parts by 2+i → (3+4i)/5
- Match each part of the diagram to the text.

Algebra and functions

- Solve linear, quadratic, rational and radical equations; check candidates in the original equation.
- $f(g(x))$ means: apply g first, then f. Usually $f(g(x)) \neq g(f(x))$.
- Transformations: $f(x-h)+k$ shifts right h and up k.
- Exponential and logarithmic functions are inverses: $\log_b y = x$ means $b^x = y$.

Geometry and trigonometry

- Use angle facts, similar triangles, Pythagoras and area and volume formulas.
- In a right triangle, $\sin$, $\cos$ and $\tan$ relate sides to angles; in any triangle, use the laws of sines and cosines.
- In coordinates: distance, midpoint and slope formulas.

Statistics, probability and modelling

- Mean, median, spread and the shape of a distribution.
- For **independent** events, $P(A \text{ and } B) = P(A)\,P(B)$.
- A model is only as good as its assumptions: state units, check the domain, and compare predictions with data.

Worked example: dividing complex numbers

- Multiply top and bottom by $1 - i$.
- Top: $(3 - 2i)(1 - i) = 3 - 3i - 2i + 2i^2 = 1 - 5i$. Bottom: $1^2 + 1^2 = 2$.
- Answer: $\frac{1}{2} - \frac{5}{2}i$. Check: $(\frac{1}{2} - \frac{5}{2}i)(1 + i) = \frac{1}{2} + \frac{5}{2} - 2i = 3 - 2i$.

Worked example: composition and powers

- $f(g(3)) = f(9) = 19$, but $g(f(3)) = g(7) = 49$: the order matters.
- $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$, and $2^{-3} = \frac{1}{8}$.
- $\sqrt{50} = 5\sqrt{2}$ is irrational, but $\sqrt{50} \cdot \sqrt{2} = \sqrt{100} = 10$ is rational.

Common errors

- Forgetting that $i^2 = -1$.
- Dividing complex numbers without the conjugate.
- Applying the functions in $f(g(x))$ in the wrong order.

Common errors (2)

- Treating 2^{-3} as negative.
- Assuming a model's prediction is certain outside the data.

Key terms

- **complex conjugate**: The number $a - bi$ for $a + bi$; their product is real.
- **composition**: Applying one function to the output of another.
- **independent**: Describes events where one does not change the probability of the other.
- **model**: A mathematical description of a real situation.

Check yourself

- Simplify $(2+3i)(2-3i)$.
- If $f(x) = x - 4$ and $g(x) = 3x$, find $g(f(6))$.
- Solve $z^2 + 16 = 0$.