Course Companion

ACT

Every topic handout in one volume

全部专题讲义合集

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Contents 目录

1. English: purpose, organisation and style 3

2. English: conventions and punctuation 7

3. Mathematics: models, functions and higher maths 11

4. Reading: details, structure and integration 14

5. Optional Science: data, investigations and models 18

6. Optional Writing: perspectives and argument 22

7. Real numbers, rational powers and complex arithmetic 27

8. Vectors and matrices as organised quantitative models 30

9. Linear, quadratic and polynomial relationships 33

10. Rational equations, radical branches and systems 36

11. Functions, composition, transformations and inverses 39

12. Sequences, finite sums, exponentials and logarithms 42

13. Trigonometric graphs, identities and general triangles 45

14. Plane transformations, coordinate reasoning and proof ... 48

15. Conics, circle relationships and geometric measurement .. 52

16. Data displays, normal distributions and association 55

17. Sampling inference, expected value and counting 58

18. Essential skills in a multi-step practical model 61

19. English: purpose, focus and argumentative support 64

20. English: paragraph order, cohesion and introductions 68

21. English: precision, concision and consistent style 72

22. English: sentence formation, agreement and punctuation . 76

23. Reading: explicit detail, inference and relationships 80

24. Reading: contextual meaning, structure and point of view 84

25. Reading: arguments, paired texts and graphic evidence .. 88

26. Science: research summaries, controls and engineering tests 92

27. Science: conflicting viewpoints and evidence that separates models 96

28. Science: data representation and defensible extrapolation 100

English: purpose, organisation and style

ACT

ACT English: Purpose, Organisation and Style

In 1946 George Orwell published an essay called “Politics and the English Language”. He argued that vague, padded writing hides what a writer means, and he gave six rules. One was: “If it is possible to cut a word out, always cut it out.” Another: “Never use a long word where a short one will do.” Orwell broke his own rules sometimes, as he admitted, but his test is still useful: does each word and sentence do a job? ACT English asks that question about a whole passage.

ACT English questions on production and style ask you to:

- Decide whether a sentence should be added or deleted, by its **relevance** 相关性 to the paragraph’s purpose.
- Judge whether a **detail** 细节 or piece of **evidence** 证据 supports a claim.
- Put sentences and paragraphs in a logical order.
- Choose concise wording in a consistent **register** 语域, avoiding **redundancy** 赘余.
- Recognise a useful **counterargument** 反论点 and the writer’s response to it.

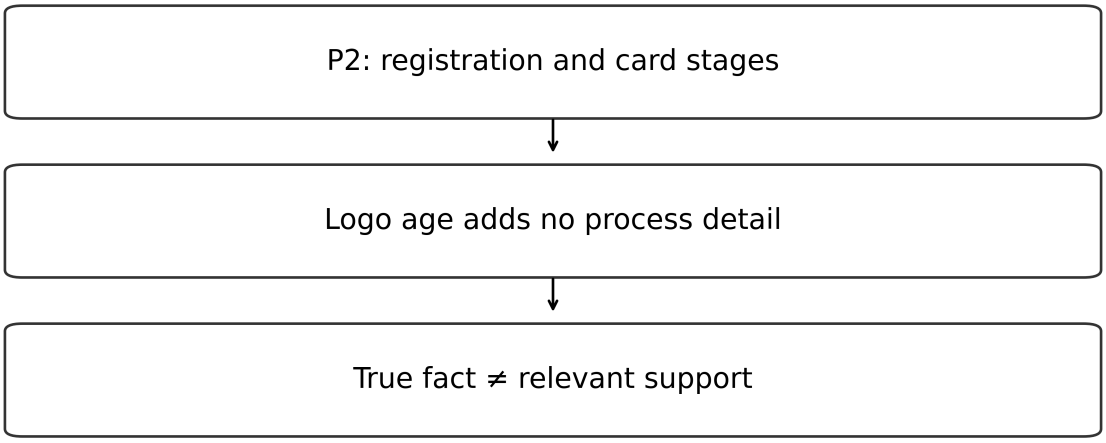
Relevance: add, delete or keep

- First name the paragraph’s purpose in a few words.
- Keep a sentence if it advances that purpose; delete it if it is true but off-topic.
- When the question asks “Should the writer add this sentence?”, the right answer gives the reason, not only yes or no.
- An objection can be relevant: a writer may raise a problem in order to answer it.

Organisation: order and placement

- Follow references: a sentence with “this method” or “these volunteers” must come after the method or the volunteers are introduced.
- Follow time order and cause before effect.
- An introduction states the topic; a conclusion returns to the main point without new details.

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Worked steps: P2: registration and card stages → Logo age adds no process detail → True fact ≠ relevant support

Style: concise and consistent

- Remove words that repeat a meaning already given: "recorded … and wrote down in their record" says one thing twice.
- Keep the passage's register. A calm, factual report should not suddenly say "totally amazing".
- Prefer the precise word: "cut complaints from six to one", not "made things better".
- The shortest option is often right, but only if it keeps the full meaning.

Practice passage

Original continuous editing passage – A place to mend

[P1] On the first Saturday of each month, a group of volunteers opens a repair table in the public library. [S1] The project began when librarian Noor noticed a row of broken desk lamps outside a nearby building. [S2] Some needed specialist work, but others had loose switches or damaged cables. [S3] Noor did not want untrained visitors to try unsafe electrical repairs. [S4] She invited a qualified repairer to explain which items the group could inspect and which should go to a professional. The first session accepted only battery-powered objects, with damaged batteries excluded. Visitors registered their items and described the faults before any work began.

[P2] At the entrance, two volunteers record each visitor's name and item. This record helps the group reunite objects with their owners. A blue card goes beside an object awaiting inspection; a white card marks one ready for collection. The cards do not certify safety or guarantee a repair. They simply show where an item is in the process. [S5] The library's logo was designed many years ago. Beside the cards, a sign asks

visitors to stay while their object is examined. Staying allows owners to explain an intermittent fault that might not appear immediately. It also gives them a chance to learn a simple maintenance habit.

[P3] Space soon became a problem. The repair table shared an alcove with readers who wanted a quiet place to work. During the first two sessions, a visitor log recorded six complaints about conversation near the alcove. Noor proposed moving the table into a meeting room. The room could hold fewer people, so a move alone would not solve the problem. The group introduced short arrival windows and kept two spaces for visitors without bookings. [S6] This totally amazing system fixed every problem forever. In fact, one visitor still waited outside, and the group had to improve the signs leading to the room. The log recorded one noise complaint during the next two sessions; attendance also fell, so the group did not attribute the whole reduction to the move.

[P4] Volunteers were careful about how they described success. An object leaving the table might work again, need another part, or require professional attention. They recorded these outcomes separately. During the trial, forty objects were inspected. Eighteen worked after a permitted minor adjustment, twelve needed a part, and ten were referred elsewhere. Counting all forty as repaired would hide important differences. [S7] The volunteers recorded the outcomes and wrote the results down in their record. Noor used the figures to decide which spare parts might be useful at later sessions. She did not use them to promise that any particular object could be repaired.

[P5] The project now has a practical aim: help visitors make an informed next decision about a broken object. For some, that means a working lamp. For others, it means knowing when further work would be unsafe or too costly. The group plans to keep the arrival windows for another trial and ask readers whether noise remains a problem. It will also compare attendance and waiting times before adding more spaces. A repair table succeeds through careful limits as well as useful repairs.

Worked example: a sentence that does not belong

Should S5 (“The library’s logo was designed many years ago”) remain in P2?

- P2’s purpose: how visitors are registered and how the cards show an item’s stage.
- The logo’s age adds nothing to that process. Delete it.
- A relevant replacement would describe the process, for example how a volunteer changes a card when an inspection finishes.

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Worked example: style and evidence

- S6, "This totally amazing system fixed every problem forever", breaks the register and is contradicted by the next sentence: one visitor still waited. A better version: "The system reduced the crowding, though not completely."
- S7 repeats itself: "The volunteers recorded the outcomes" is enough.
- In P3, the six recorded complaints are the evidence that noise was a problem. The sentence saying the meeting room holds fewer people is a relevant objection that explains why the group chose arrival windows; keep it.

Common errors

- Keeping a true sentence that does not serve the paragraph.
- Choosing "yes" or "no" for an add/delete question without checking the reason given.
- Placing a sentence before the thing it refers to.
- Choosing a longer option that repeats a meaning.
- Accepting an exaggeration that the passage contradicts.

Key terms

relevance: How well a sentence serves the purpose of its paragraph.

detail: A specific fact or example.

evidence: Information that supports a claim.

register: The level of formality of a piece of writing.

redundancy: Saying the same thing twice without need.

counterargument: A point against the writer's position.

Check yourself

- A paragraph explains how bees find flowers. Should it include "Bees appear on many company logos"?
- Make concise: "The meeting was postponed until a later date."
- Where must the sentence "This change saved time" go?

Answers: no: it is true but irrelevant to how bees find flowers. "The meeting was postponed." After the sentence that describes the change.

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English: conventions and punctuation

ACT

ACT English: Conventions and Punctuation

On 22 July 1962 the American space probe Mariner 1, bound for Venus, veered off course shortly after launch, and a safety officer destroyed it less than five minutes into the flight. Investigators traced the fault to the guidance software, often explained as a single missing mark in a handwritten formula that had been typed into the program. The writer Arthur C. Clarke later called it “the most expensive hyphen in history”. Most punctuation mistakes cost less, but on ACT English they cost marks, and they can change meaning just as completely.

ACT questions on the **conventions** 语言规范 of standard English ask you to:

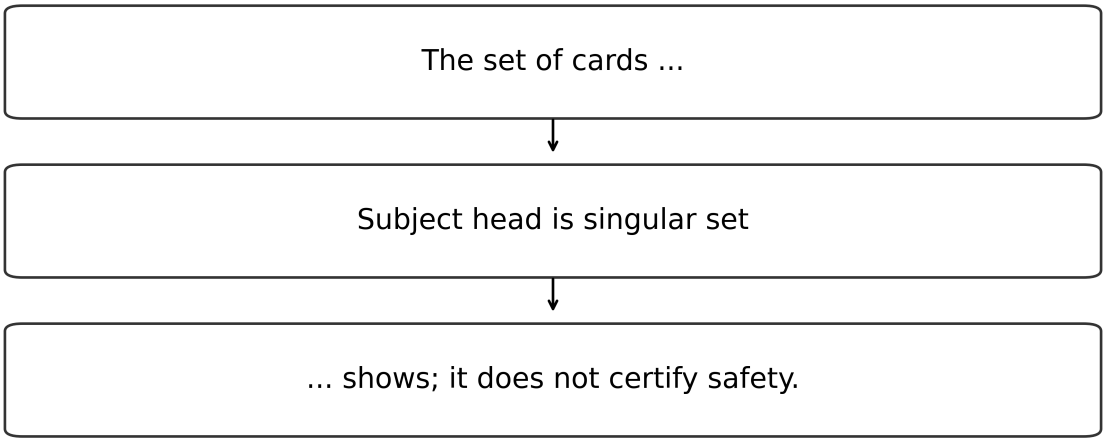
- Mark sentence boundaries and avoid a **comma splice** 逗号拼接错误.
- Make verbs and pronouns show **agreement** 一致关系 with their subjects and nouns.
- Place each **modifier** 修饰语 next to the word it describes.
- Keep lists and comparisons in **parallelism** 平行结构.
- Use commas, apostrophes, colons and dashes correctly.

Sentence boundaries

- An independent clause has a subject and verb and can stand alone.
- Join two independent clauses with a full stop, a semicolon, or a comma plus and, but, or, so, yet, for or nor.
- A comma alone between them is a comma splice; nothing at all is a run-on.

Agreement

- A verb agrees with its true subject, not the nearest noun: “The set of blue cards shows the stage.”
- A pronoun agrees with the noun it replaces: “The group updates its records.”
- Keep verb tense consistent unless the time changes.



Worked steps: The set of cards … → Subject head is singular set → … shows; it does not certify safety.

Modifiers and parallelism

- An opening phrase describes the subject that follows the comma: “Inspecting the lamp, Noor noticed…”.
- Items in a list share one form: “to inspect, record and advise”.
- Compare like with like.

Punctuation marks

- Commas: after an introductory clause; around nonessential information (in a matching pair); between items in a list.
- No comma between a subject and its verb, or between a verb and its object.
- Apostrophes: “its” is possessive; “it’s” means “it is”. Plural possessive: “the volunteers’ table”.
- A colon follows a complete clause; it introduces a list, explanation or example.

Practice passage

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Worked example: agreement and boundary

Repair: "The set of blue cards show the current stage, they do not certify safety."

- Subject: "set", singular, so "shows".
- Two independent clauses joined by a comma alone: a comma splice.
- Corrected: "The set of blue cards shows the current stage; it does not certify safety." The pronoun "it" now matches "set".

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Worked example: modifier, parallelism and apostrophes

- "Inspecting the lamp, a loose switch was noticed by Noor." The switch did not inspect anything. Corrected: "Inspecting the lamp, Noor noticed a loose switch."
- "The group aims to inspect, recording, and advise." Corrected: "to inspect, record, and advise".
- "The group updates _____ records": "its" (possession). "_____ important to label outcomes": "It's" (it is). Test by reading "it is" aloud.

Common errors

- Joining independent clauses with a comma alone.
- Matching the verb to the nearest noun.
- Writing "it's" for possession.
- Leaving an opening phrase with no subject to describe.
- Putting a comma between a subject and its verb.

Key terms

conventions: The accepted rules of grammar, usage and punctuation.

comma splice: Two independent clauses joined only by a comma.

agreement: Matching a verb to its subject, or a pronoun to its noun, in number.

modifier: A word or phrase that describes another word.

parallelism: Using the same grammatical form for items in a list or comparison.

Check yourself

- Correct: "The list of names were long."
- Choose: "The company changed _____ logo." (its / it's)
- Correct: "Running for the bus, my bag fell open."

Answers: "was long". "its". "Running for the bus, I felt my bag fall open" (or another subject who runs).

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Mathematics: models, functions and higher maths

ACT

ACT Mathematics: Models, Functions and Higher Maths

In 1572 the Italian engineer Rafael Bombelli published *L'Algebra*. Cardano's method for cubic equations, printed 27 years earlier, sometimes produced square roots of negative numbers even when the answer was an ordinary number, and most mathematicians simply gave up at that point. Bombelli wrote down rules for calculating with these strange quantities, and showed they could lead to the real answer. They became the complex numbers. ACT Mathematics covers the whole range from such number rules to functions, geometry, statistics and real-world models; this overview maps it.

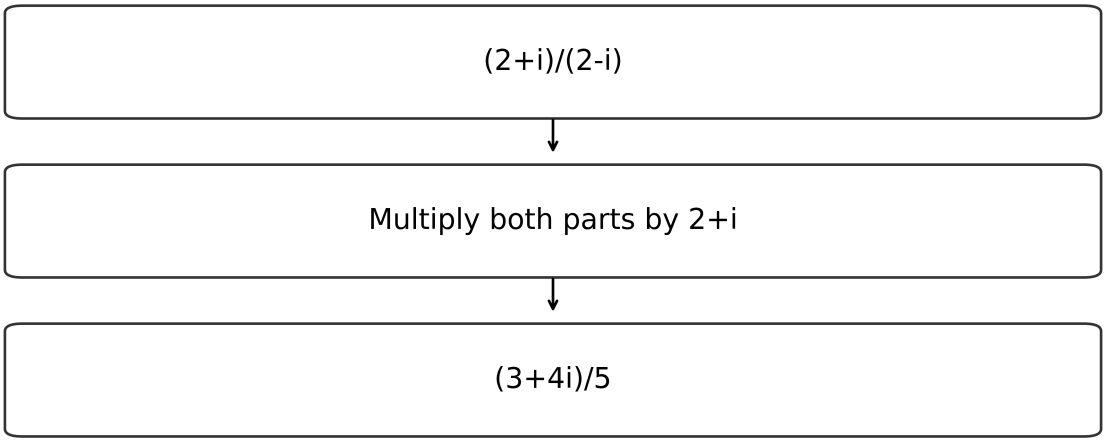
ACT Mathematics questions ask you to:

- Calculate with real and complex numbers, using the **complex conjugate** 复共轭 to divide.
- Work with functions, including **composition** 函数复合.
- Solve problems in geometry and trigonometry.
- Use probability, including **independent** 独立的 events, and statistics.
- Build and use a **model** 模型 of a real situation.

Number and complex numbers

- $i^2 = -1$. Add and multiply complex numbers like binomials, replacing i^2 with -1.
- To divide, multiply top and bottom by the conjugate of the denominator: the conjugate of $a + bi$ is $a - bi$.
- $(a + bi)(a - bi) = a^2 + b^2$, a real number.
- Rational powers: $x^{m/n} = (\sqrt[n]{x})^m$; negative powers: $x^{-n} = 1/x^n$.

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Worked steps: (2+i)/(2-i) → Multiply both parts by 2+i → (3+4i)/5

Algebra and functions

- Solve linear, quadratic, rational and radical equations; check candidates in the original equation.
- $f(g(x))$ means: apply g first, then f. Usually $f(g(x)) \neq g(f(x))$.
- Transformations: $f(x-h)+k$ shifts right h and up k.
- Exponential and logarithmic functions are inverses: $\log_b y = x$ means $b^x = y$.

Geometry and trigonometry

- Use angle facts, similar triangles, Pythagoras and area and volume formulas.
- In a right triangle, sin, cos and tan relate sides to angles; in any triangle, use the laws of sines and cosines.
- In coordinates: distance, midpoint and slope formulas.

Statistics, probability and modelling

- Mean, median, spread and the shape of a distribution.
- For independent events, $P(A \text{ and } B) = P(A)\,P(B)$.
- A model is only as good as its assumptions: state units, check the domain, and compare predictions with data.

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Worked example: dividing complex numbers

Write $\dfrac{3-2i}{1+i}$ in the form $a+bi$.

- Multiply top and bottom by $1-i$.
- Top: $(3-2i)(1-i) = 3-3i-2i+2i^2 = 1-5i$. Bottom: $1^2+1^2 = 2$.
- Answer: $\frac{1}{2}-\frac{5}{2}i$. Check: $(\frac{1}{2}-\frac{5}{2}i)(1+i) = \frac{1}{2}+\frac{5}{2}-2i = 3-2i$.

Worked example: composition and powers

Let $f(x) = 2x+1$ and $g(x) = x^2$.

- $f(g(3)) = f(9) = 19$, but $g(f(3)) = g(7) = 49$: the order matters.
- $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$, and $2^{-3} = \frac{1}{8}$.
- $\sqrt{50} = 5\sqrt{2}$ is irrational, but $\sqrt{50}\cdot\sqrt{2} = \sqrt{100} = 10$ is rational.

Common errors

- Forgetting that $i^2 = -1$.
- Dividing complex numbers without the conjugate.
- Applying the functions in $f(g(x))$ in the wrong order.
- Treating 2^{-3} as negative.
- Assuming a model's prediction is certain outside the data.

Key terms

complex conjugate: The number $a-bi$ for $a+bi$; their product is real.

composition: Applying one function to the output of another.

independent: Describes events where one does not change the probability of the other.

model: A mathematical description of a real situation.

Check yourself

- Simplify $(2+3i)(2-3i)$.
- If $f(x) = x-4$ and $g(x) = 3x$, find $g(f(6))$.
- Solve $z^2+16 = 0$.

Answers: $4+9 = 13$. $g(2) = 6$. $z = 4i$ or $z = -4i$.

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Reading: details, structure and integration

ACT

ACT Reading: Details, Structure and Integration

In 1879 the French eye doctor Louis Émile Javal noticed something surprising about reading. The eyes do not glide smoothly along a line of print; they jump, in quick movements later called saccades, and pause briefly on some words. Later research showed that readers take in information mostly during those pauses, and that skilled readers sometimes move back to check. Good reading is active: it stops on the words that matter. ACT Reading rewards the same habit of locating and checking the exact words.

ACT Reading questions ask you to:

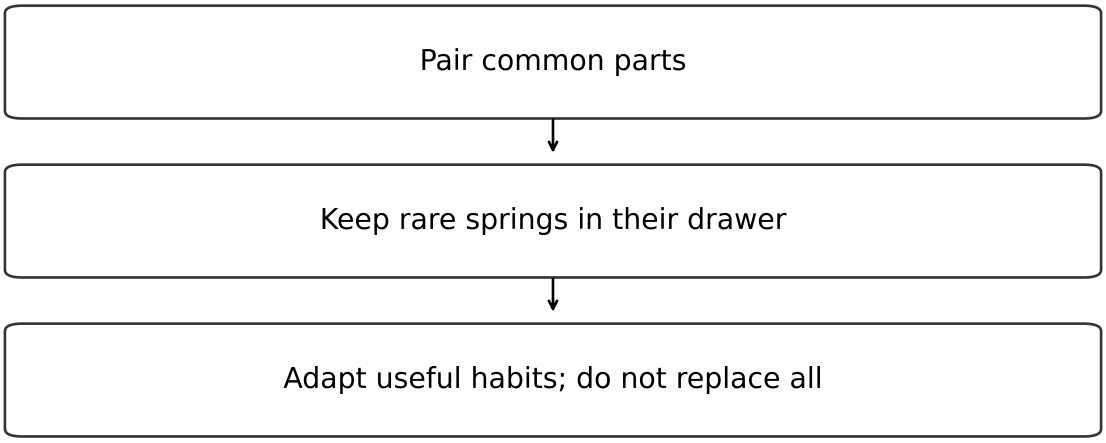
- Find an explicit **detail** 细节 and make an **inference** 推断 from it.
- Describe how a passage is organised and what a part of it does.
- Identify the narrator's or author's **viewpoint** 观点 and **tone** 语气.
- Support each answer with **evidence** 证据 from the passage, and integrate information from two texts or a graphic.

Details and inferences

- For a detail question, go back to the passage and find the line; do not answer from memory.
- An inference joins stated facts. The right answer can be pointed to in the text.
- Reject answers that are true in life but not in the passage, or that add a number or a cause.

Structure and function

- Ask what each paragraph does: introduces a situation, describes a problem, shows a change, reflects.
- A detail in a story often has a job: to show a feeling or a relationship without stating it.
- Look at how the ending connects back to the beginning.



Worked steps: Pair common parts → Keep rare springs in their drawer → Adapt useful habits; do not replace all

Viewpoint and tone

- Who is speaking? In a first-person story, the narrator's knowledge is limited; they may be unsure of others' feelings.
- Tone is the attitude shown by word choice: affectionate, uncertain, critical, amused.
- An answer that makes the tone more extreme than the words is usually wrong.

Integrating information

- In paired passages, summarise each text first, then compare.
- With a graphic, read its title, labels and units, and check that the answer uses the right part of it.

Practice passage

Original literary passage – The spare key

When I returned to the workshop after my uncle retired, the key felt too small for the responsibility attached to it. He had left the drawers neatly labelled, as though the labels might continue his explanations in his absence. I turned on the overhead light and read "washers", "pins", "springs". Each word named an orderly compartment. The workbench, however, carried a shallow circular mark where his tea cup had stood, and no label could explain why that empty circle held my attention longer than the stocked shelves.

A neighbour arrived with a bent gate latch. I found a replacement pin quickly, then spent several minutes searching for the washer that would keep it in place. My uncle had sorted objects by size. He could see a repair in his mind before he opened a

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drawer; I could not. The neighbour waited without complaint, which made the delay feel more noticeable rather than less. At last I found the washer, fitted the pin and watched the latch swing freely. The repair was simple. Finding its parts had not been.

That evening I placed commonly paired pieces in small trays. The shelves lost some of their symmetry. I imagined my uncle raising an eyebrow, then remembered how often he had moved a tool after noticing where his hand expected it to be. His neat labels had been the result of change, not a prohibition against it. I left the rare springs in their old drawer because they had no regular partners. There was no reason to change them merely to prove that the workshop was mine.

A week later my uncle visited. He opened a tray and asked where the larger washers had gone. I showed him the remaining size-labelled drawer. "So you have kept both systems," he said. I could not tell whether his smile meant approval or amusement at my earnestness. He tested a latch, set it down, and asked for tea. I placed his cup on the old mark without thinking. Only after he left did I notice that I had made room beside it for my own.

I had expected taking charge to mean replacing his habits with mine. Instead, I was learning which habits solved a problem and which had depended on a skill I had not yet acquired. The workshop did not need a dramatic beginning. It needed a washer that could be found, a safe repair, and enough empty space for the next person to work.

Worked example: a detail and its purpose

Why does the narrator start pairing parts?

- The problem: a simple latch repair is delayed while the narrator searches for the matching washer (paragraph 2).
- The uncle could "see a repair in his mind"; the narrator could not. Pairing common parts helps someone without that skill.
- The passage gives no measured time saved, so the answer should describe the purpose, not a number.
- Not everything changed: the rare springs stay in their old drawer, and a size-labelled drawer remains. The narrator "kept both systems".

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Worked example: interpreting the ending

What do the two cup places at the end suggest?

- The narrator puts the uncle's cup on the old mark "without thinking", then sees that they "had made room beside it" for their own.
- A supported reading: the narrator is building a role of their own while keeping the relationship with the uncle.
- Unsupported: that the uncle demanded a permanent desk. Nothing like this is reported.
- The final paragraph confirms the theme: not replacing habits, but learning which ones to keep.

Common errors

- Answering from memory instead of the line.
- Choosing an inference that adds a number or a cause.
- Treating a narrator's guess as a fact ("I could not tell whether his smile meant approval").
- Making the tone more extreme than the passage.
- Choosing an answer that fits one paragraph but not the whole passage.

Key terms

detail: A specific piece of information stated in the passage.

inference: A conclusion that the stated information supports.

viewpoint: The position from which a story is told or an argument made.

tone: The attitude that word choice shows.

evidence: Words from the passage that support an answer.

Check yourself

- The narrator says "I could not tell whether his smile meant approval or amusement." Can you answer that the uncle approved?
- What is the function of the circular tea-cup mark in paragraph 1?
- Which word best describes the narrator's tone at the end: bitter, reflective or triumphant?

Answers: no: the narrator does not know. It shows the narrator's feeling for the uncle, beyond the labelled order. Reflective.

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Optional Science: data, investigations and models

ACT

ACT Science: Data, Investigations and Models

On 5 May 1881, at a farm at Pouilly-le-Fort near Paris, Louis Pasteur began a public test of his anthrax vaccine. Of about 50 sheep, half were vaccinated and half were not; later, all were injected with live anthrax. By 2 June almost all the unvaccinated sheep were dead, while the vaccinated ones were healthy. Journalists and farmers watched. The test convinced people because it compared two groups treated the same way in every respect but one. ACT Science is about reading such investigations: their data, their design and the models that explain them.

ACT Science questions ask you to:

- Read data from tables and graphs, with units.
- Identify the changed, measured and controlled **variable** 变量 of an experiment, and the **control group** 对照组.
- Judge whether a **controlled comparison** 控制比较 isolates one factor.
- Compare models and decide what **evidence** 证据 could separate them.
- Apply **engineering design** 工程设计 criteria and evaluate a **proposal** 提案.

Reading data

- Read the title, axes, units and key before any number.
- Describe a trend in words: increases, decreases, levels off.
- Interpolate between measured values with care; extrapolate beyond them only with a stated assumption.

Experimental design

- The independent variable is changed on purpose; the dependent variable is measured; controlled variables are kept the same.
- A control group receives no treatment, for comparison.
- A fair test changes one factor at a time. Repeated trials show how much results vary.

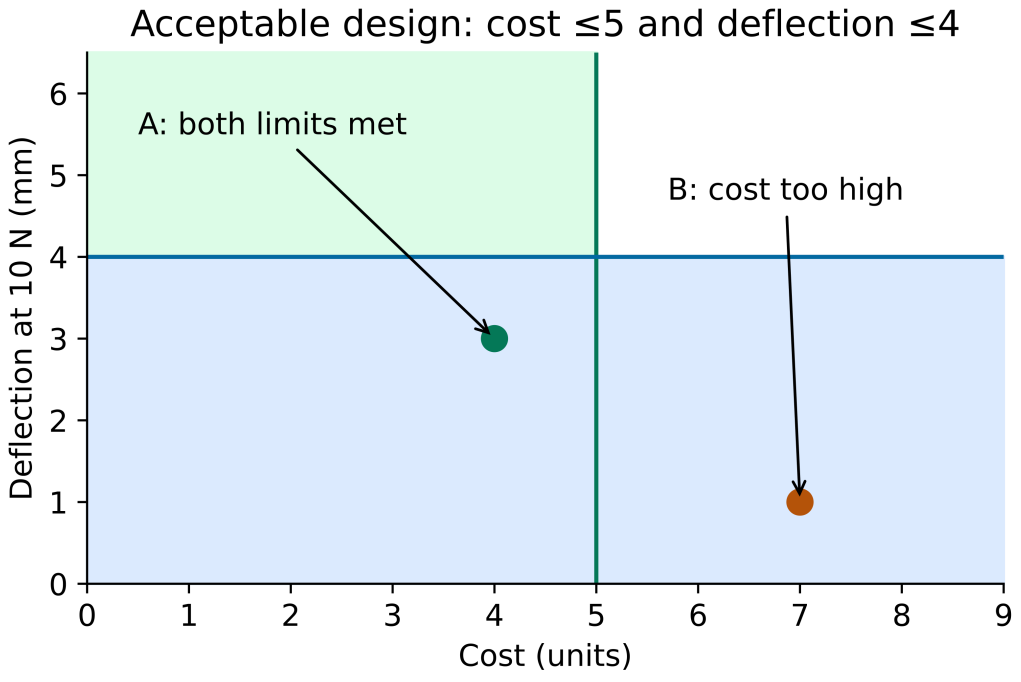
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Models and viewpoints

- Two models can both explain the same trend. To separate them, find a situation where they predict different results.
- A result can be consistent with a model without proving it.

Design criteria

- An engineering design must meet every stated requirement, such as cost and performance, not just the most impressive one.
- A design inside all limits is acceptable; one outside any limit is not.



A design is acceptable only inside both limits: cost at most 5 and deflection at most 4

Practice research summary

Original hypothetical research summary – A cooling sleeve

A laboratory team tests sleeves intended to reduce a small sensor's temperature during operation. All sensors start at 22°C. Temperature is measured after ten minutes of use with the same power setting, room conditions and measuring instrument. Sleeve A costs 4 units, B costs 7 units, and an unsleeved sensor is the control. The design brief requires a cost no greater than 5 units and a final temperature no greater than 35°C. Lower temperature alone is not the only design criterion.

Experiment 1 uses identical sensors and records three independent trials per sleeve. Final temperatures in °C are: control 41,42,43; A 33,34,35; B 30,31,32. The team

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resets the starting temperature and checks the sensor charge before each trial. They report the means and the full observed ranges. The ranges describe these trials, not every future result. The team has not yet measured sleeve durability.

Experiment 2 investigates thickness for material A. Each sleeve has the same length and fit; thicknesses are 1,2,3 mm. Mean final temperatures are 38,34,32°C respectively. All other stated conditions match Experiment 1. Changing thickness also changes material quantity, so a future design review must measure costs instead of assuming they stay at 4 units. A student proposes testing a 3-mm sleeve in material B against a 1-mm sleeve in material A to identify thickness's effect. This comparison would change two factors at once.

Two original models explain the observed temperature pattern. Model P says final temperature depends only on thickness, regardless of material. Model Q says both thickness and material matter; at equal thickness it predicts that material B produces a lower final temperature than A under the same conditions. Both models predict lower final temperatures as thickness rises over the tested 1–3 mm interval. A decreasing trend alone therefore cannot separate them.

Experiment 3 tests new A and B sleeves, each 2 mm thick, in matched conditions. Mean final temperatures are A 34°C and B 31°C. The measuring instrument's stated resolution is 0.1°C, and the team checks its calibration with a reference. Repeated trials would still be needed to characterise variability and rule out other differences in manufacture. The result conflicts with P's material-independence prediction and is consistent with Q's direction; it is not proof that Q is the only possible explanation.

A separate time series for one sensor with sleeve A records temperatures 22,28,32,34°C at 0,2,4,6 minutes. These observations show warming with successively smaller two-minute increases. No measurements beyond six minutes are supplied in this series. A proposal to continue the first interval's slope to twelve minutes is a model assumption that later measurements may contradict, not a reading from the table.

Worked example: meeting the design brief

Use Experiment 1.

- Means: control $(41 + 42 + 43)/3 = 42$ °C; A = 34 °C; B = 31 °C. Ranges: 41–43, 33–35, 30–32 °C.
- A costs 4 units (at most 5) and reached at most 35 °C: it meets both limits in these trials.
- B is coolest but costs 7 units, more than 5, so it fails the brief.
- Durability has not been measured, so even A is not yet fully approved.

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Worked example: a fair comparison

- In Experiment 2, thickness is changed and final temperature is measured; material, length, fit, start temperature and conditions are controlled.
- The student's proposal (3 mm of B against 1 mm of A) changes material and thickness together, so a difference could come from either. Repeating it would not fix this.
- Models P and Q both predict cooler results with thicker sleeves, so Experiment 2 cannot separate them. Experiment 3 holds thickness at 2 mm and changes only material: A 34 °C, B 31 °C. This conflicts with P and is consistent with Q, without proving Q.

Common errors

- Reading numbers before units and labels.
- Choosing the best-performing design while ignoring a cost limit.
- Accepting a comparison that changes two factors at once.
- Treating "consistent with a model" as "proves the model".
- Extending a trend far beyond the data without saying so.

Key terms

variable: A quantity that can change in an experiment.

control group: A group that receives no treatment, used for comparison.

controlled comparison: A comparison in which only one factor differs.

evidence: Data that support or weaken a claim.

engineering design: Creating a solution that meets stated requirements and limits.

proposal: A suggested plan or test.

Check yourself

- In Experiment 2, which variable is independent and which is dependent?
- Which result could tell Models P and Q apart?
- The time series reaches 34 °C at 6 minutes. Can the table tell you the temperature at 12 minutes?

Answers: thickness; final temperature. Two materials at equal thickness, as in Experiment 3. No: that would be an extrapolation based on an assumption.

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Optional Writing: perspectives and argument

ACT

ACT Writing: Perspectives and Argument

Between August and October 1858 two candidates for the US Senate, Abraham Lincoln and Stephen Douglas, held seven public debates across Illinois. Each lasted about three hours: one man spoke for an hour, the other replied for an hour and a half, and the first had half an hour to answer. Crowds of thousands stood to listen. The format forced each speaker to deal with the other's actual arguments, not just repeat his own. The optional ACT Writing test asks for the same discipline in 40 minutes: take a position, and engage seriously with other perspectives.

The ACT Writing task asks you to:

- State a clear position, your **thesis** 论点, on an issue.
- Analyse how your view relates to each given **perspective** 视角.
- Develop an **argument** 论证 with reasons and **evidence** 证据, such as examples.
- Answer a **counterargument** 反论点, and make a **concession** 让步 where a point is fair.

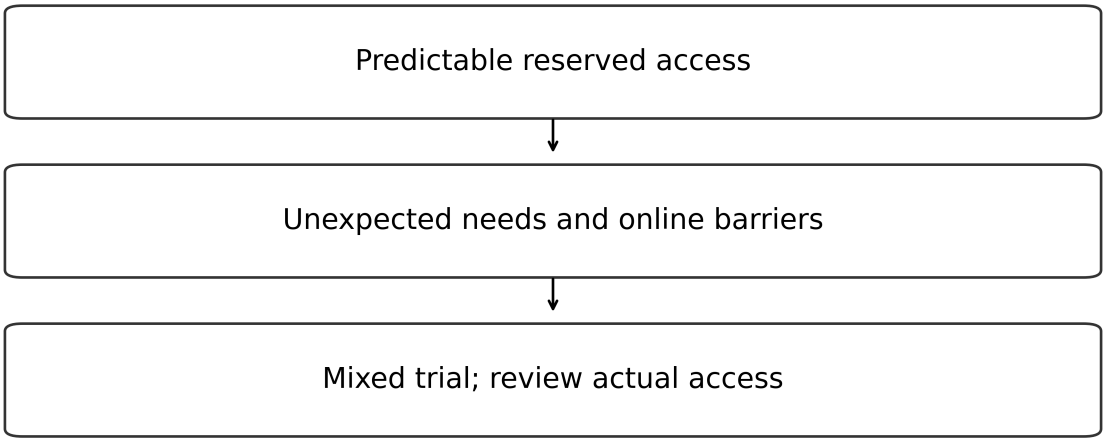
Planning a position

- Read the issue and the three perspectives. Note what each values: fairness, efficiency, freedom, safety.
- Your position can agree with one, combine parts of several, or offer a new view.
- Write one sentence that states it and the main reason.

Engaging the perspectives

- For each perspective, say where you agree, where you disagree, and why.
- Do not describe two perspectives as if they say the same thing.
- A strong essay shows the conditions under which a view works or fails.

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Worked steps: Predictable reserved access → Unexpected needs and online barriers → Mixed trial; review actual access

Developing reasons

- Each body paragraph makes one point and explains it with an example or a likely consequence.
- Imagined examples can show how something would work; do not present them as measured facts.
- Link paragraphs with clear transitions.

Concession and conclusion

- A concession admits a fair point against you, then shows why your position still holds, or how it should change.
- The conclusion restates the position with its limits; it adds no new argument.

The practice issue

Writing issue. A community learning centre has a small number of shared tools. Should every use require advance booking?

Perspective 1: Requiring bookings for all tools makes access predictable and prevents visitors travelling only to find everything occupied.

Perspective 2: Walk-in access is important because people may discover a need unexpectedly or be unable to book online.

Perspective 3: A mixed system can reserve some capacity while retaining walk-in access, but staff must explain and administer the rules.

Write a sustained argument stating your own position, analysing its relationship with

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all three perspectives, and developing reasons and examples. This is original classroom practice, not an official prompt, timed form or score prediction. No official word minimum is asserted here.

Worked example: a plan

- Position: a mixed system, with some sessions bookable and some kept for walk-in visitors.
- Perspective 1: agree that predictable access matters; reject booking for every use.
- Perspective 2: accept the concern about unexpected needs and online barriers; question whether an unrestricted queue is fair.
- Perspective 3: develop its compromise, and take seriously its warning about running the rules.

Worked example: a model essay

A learning centre should make access predictable without requiring every visitor to predict every need. I would reserve some tool sessions in advance and keep others available to people who arrive without bookings. This position accepts the first perspective's concern about wasted journeys, the second perspective's concern about unexpected needs, and the third perspective's warning that a compromise requires clear administration. Its success would depend on practical rules rather than the word mixed alone.

Advance booking has an important benefit. Imagine a visitor who needs a particular tool for a supervised project and must travel across town. Knowing that a session is reserved lets the visitor organise transport and bring the right materials. A queue discovered on arrival may waste both the journey and the preparation. The first perspective therefore identifies a real access problem. However, reserving every minute would solve that problem by creating another: someone whose tool breaks unexpectedly might find a room apparently full of reservations, even when several booked users do not arrive. Predictability should be protected, but it should not be confused with maximum use of every tool.

The second perspective is strongest when it focuses on barriers rather than simply celebrating spontaneity. A visitor may not have reliable internet access, may struggle with an online form, or may discover a missing tool only while working. A rule requiring online reservations would treat these different situations as failures to plan. Yet unrestricted walk-in access also has limits. The same confident regular visitors could take the earliest spaces repeatedly, while a newcomer waits without knowing when a turn will come. Removing bookings would not automatically make access fair. The centre needs a visible queue and staff assistance as well as open spaces.

A mixed system can address both sets of concerns. For an initial trial, the centre

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could offer bookable sessions and a clearly marked number of walk-in sessions for each heavily used tool. Reservations should also be possible in person or by telephone, so that online access does not become a condition of participation. A short grace period could release an unused reservation, provided the rule is stated when the booking is made. Visitors would then know both what a reservation protects and when it expires. These are proposed trial rules, not claims that one proportion of spaces suits every centre.

Administration is a serious objection to this approach. Staff already supervise safe tool use, and a complicated allocation system could distract them. This is where the third perspective places a condition on its own compromise. The centre should start with a simple timetable for only its busiest tools, use one shared booking record, and display the next available session. If staff cannot apply a rule consistently, they should simplify it. Special exceptions that depend on who happens to be at the desk would undermine the fairness the system is intended to improve.

The trial should be judged by more than the number of reservations. Staff could record unused booked sessions, waiting times, visitors turned away, and requests for help with booking. They should also ask whether newcomers and people without internet access can obtain a turn. If unused reservations remain common, more walk-in capacity may be needed. If long journeys frequently end in disappointment, more reserved capacity may be justified. The recommendation thus concedes that the balance should change with evidence rather than remain fixed for the sake of consistency.

Shared tools exist to support learning, not to reward either perfect planning or early arrival alone. A clearly explained mixed system preserves useful certainty while leaving room for unexpected needs. Its limits must remain visible: alternative booking routes, manageable staff work, and a review of actual access. Those conditions turn a compromise into a policy that people can understand and the centre can revise.

Evaluating the model

- It relates its position to all three perspectives, accepting part of each and rejecting what goes too far.
- Its examples show how things would work and are not presented as measured results.
- Its concession is real: the balance of sessions should change with evidence.
- Before acting on it, a centre would need local facts such as staffing, demand and safety rules.

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Common errors

- Summarising the perspectives without taking a position.
- Agreeing with all three without explaining how they fit together.
- Making claims with no reasons or examples.
- Inventing statistics and presenting them as facts.
- A conclusion that only repeats the introduction.

Key terms

thesis: The main position the essay argues.

perspective: A point of view on an issue.

argument: A position supported by reasons.

evidence: Examples or facts that support a reason.

counterargument: A point against your position.

concession: Admitting that an opposing point is partly right.

Check yourself

- What must your essay do with the given perspectives besides summarise them?
- Is "Studies show 80% of visitors prefer booking" acceptable if you have no such study?
- What does a concession do?

Answers: analyse how each relates to your own position. No: present imagined cases as examples, not as data. It admits a fair point against you and shows how your view holds or changes.

Real numbers, rational powers and complex arithmetic

ACT

Euclid's *Elements*, written in Alexandria around 300 BC, includes the classic argument that no fraction squared gives exactly 2: $\sqrt{2}$ is irrational. For centuries afterwards, square roots of negative numbers were thought impossible. In 1777 Leonhard Euler began writing i for $\sqrt{-1}$, and by the early 1800s mathematicians such as Gauss treated complex numbers as points in a plane. ACT questions move across this whole number system: fractions, roots, powers and complex numbers.

These questions ask you to:

- Classify numbers as a **rational number** 有理数 or an **irrational number** 无理数.
- Use the rules for integer and rational powers, and simplify roots.
- Calculate with the **imaginary unit** 虚数单位 i and complex numbers.
- Divide complex numbers using the **complex conjugate** 复共轭.

Rational and irrational numbers

- A rational number can be written as a fraction of integers; its decimal ends or repeats.
- $\sqrt{2}$, $\sqrt{3}$ and π are irrational.
- Sums and products of irrational numbers can be rational: $\sqrt{2}\cdot\sqrt{2}=2$ and $\sqrt{2}+(-\sqrt{2})=0$. So test examples instead of assuming.

Powers and roots

$$x^{-n}=\frac{1}{x^n} \qquad x^{m/n}=\left(\sqrt[n]{x}\right)^m \qquad x^a x^b = x^{a+b}$$

- A negative power gives a reciprocal, not a negative number: $2^{-3}=\frac{1}{8}$.
- Simplify a root by taking out square factors: $\sqrt{50}=\sqrt{25\cdot 2}=5\sqrt{2}$.
- $\sqrt{x^2}=|x|$, so $\sqrt{(-7)^2}=7$.

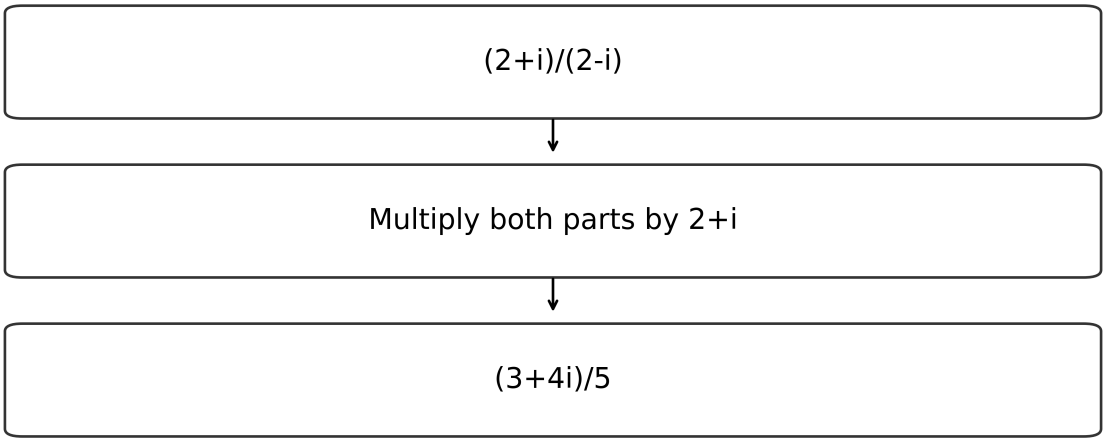
Complex numbers

- $i^2=-1$, and powers of i repeat every four: $i,-1,-i,1$.
- A complex number $a+bi$ has real part a and imaginary part b.
- Add by combining like parts; multiply like brackets and replace i^2 with -1.
- $z^2=-16$ has the two solutions $z=4i$ and $z=-4i$.

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Dividing with the conjugate

- The conjugate of $a+bi$ is $a-bi$, and $(a+bi)(a-bi)=a^2+b^2$, a real number.
- To divide, multiply the top and bottom by the conjugate of the bottom.



Worked steps: (2+i)/(2-i) → Multiply both parts by 2+i → (3+4i)/5

Worked example: a quotient

Write $\dfrac{3-2i}{1+i}$ in the form $a+bi$.

- Multiply by $\dfrac{1-i}{1-i}$.
- Top: $(3-2i)(1-i)=3-3i-2i+2i^2=1-5i$. Bottom: $1+1=2$.
- Answer: $\frac{1}{2}-\frac{5}{2}i$.
- Check: $(\frac{1}{2}-\frac{5}{2}i)(1+i)=\frac{1}{2}+\frac{1}{2}i-\frac{5}{2}i+\frac{5}{2}=3-2i$.

Worked example: classifying and evaluating

- $\sqrt{50}=5\sqrt{2}$: irrational. $\sqrt{50}\cdot\sqrt{2}=\sqrt{100}=10$: rational.
- $\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$: irrational. So "irrational times irrational" can go either way.
- $32^{2/5}=(\sqrt[5]{32})^2=2^2=4$, and $2^{-3}=\frac{1}{8}$.

Common errors

- Treating a negative power as a negative number.
- Writing $\sqrt{(-7)^2}=-7$.
- Forgetting that $i^2=-1$ when multiplying.
- Dividing complex numbers by splitting the denominator.

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- Assuming the product of two irrational numbers is irrational.

Key terms

rational number: A number that can be written as a fraction of two integers.
irrational number: A real number that cannot be written as such a fraction.
imaginary unit: The number i, with $i^2 = -1$.
complex conjugate: For $a + bi$, the number $a - bi$.

Check yourself

- Find i^7.
- Simplify $\sqrt{72}$.
- Add $(2 + i) + (3 - 4i)$, then multiply $(1 + i)(1 - i)$.

Answers: $i^7 = i^4 \cdot i^3 = -i$. $6\sqrt{2}$. $5 - 3i$; 2.

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Vectors and matrices as organised quantitative models

ACT

In 1858 the English mathematician Arthur Cayley, then working as a lawyer in London, published "A Memoir on the Theory of Matrices". He treated a rectangular table of numbers as a single object that could be added and multiplied, and noticed that, unlike ordinary numbers, the order of multiplication matters. A century later, matrices were running in computers to model economies, structures and graphics. On the ACT, vectors and matrices appear as tidy ways to organise quantities.

These questions ask you to:

- Find a **displacement vector** 位移向量 and its **magnitude** 模.
- Add vectors and multiply them by a **scalar** 标量.
- Use a **matrix** 矩阵 to organise data, and multiply matrices when the sizes allow.
- Calculate a **dot product** 点积.

Vectors

- A vector has size and direction. In components, the displacement from A to B is $B - A$.
- Its magnitude is $\sqrt{x^2 + y^2}$, by Pythagoras.
- Add vectors component by component; a scalar multiplies every component.
- The direction matters: $\overrightarrow{BA} = -\overrightarrow{AB}$.

Matrices: size and simple operations

- A matrix with m rows and n columns has size $m \times n$.
- Add or subtract only matrices of the same size, entry by entry.
- A scalar multiplies every entry.

Matrix multiplication

- AB is defined only if the number of columns of A equals the number of rows of B. An $m \times n$ times an $n \times p$ gives $m \times p$.
- Each entry is a row of A times a column of B: multiply matching entries and add.
- In general $AB \neq BA$, and one may exist while the other does not.

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Quantity rows × price column (4, 5)

Row	Item A	Item B	Total cost
List 1	2	1	13
List 2	1	3	19

Quantity rows times the price column give each total; here the prices are 4 and 5

Dot product and determinant

$$(a,b)\cdot(c,d)=ac+bd \qquad \det\begin{pmatrix}a & b\\ c & d\end{pmatrix}=ad-bc$$

- A dot product of zero means the vectors are perpendicular.
- A 2×2 matrix with determinant zero has no inverse.

Worked example: displacement

Points A and B, in metres, are $(1,-2)$ and $(7,6)$.

- $\overrightarrow{AB}=(7-1,\ 6-(-2))=(6,8)$ m.
- Magnitude: $\sqrt{36+64}=10$ m.
- Adding the displacement $(-3,2)$ m to B gives the point $(4,8)$.
- Dot product $(6,8)\cdot(-3,2)=-18+16=-2$: not zero, so not perpendicular.

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Worked example: an order matrix

Two orders contain $(3, 2)$ and $(1, 4)$ pens and notebooks. Pens cost 2 yuan and notebooks 7 yuan.

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 3(2) + 2(7) \\ 1(2) + 4(7) \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$$

- The orders cost 20 yuan and 30 yuan. The items must be in the same order in the rows and in the price column.
- If A is 2×3 and B is 3×4, then AB is 2×4, but BA is undefined (4 columns against 2 rows).

Common errors

- Subtracting the points in the wrong order.
- Adding magnitudes instead of components.
- Multiplying matrices entry by entry.
- Assuming $AB = BA$.
- Mismatching the item order between rows and columns.

Key terms

displacement vector: The change in position from one point to another.

magnitude: The length of a vector.

scalar: An ordinary number that multiplies a vector or matrix.

matrix: A rectangular array of numbers.

dot product: The sum of the products of matching components.

Check yourself

- Find the magnitude of $(5, 12)$.
- Find $2(1, -3) + (4, 1)$.
- What is the size of a 3×2 matrix times a 2×5 matrix?

Answers: 13. $(6, -5)$. 3×5.

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Linear, quadratic and polynomial relationships

ACT

A clay tablet in the British Museum, catalogued BM 13901 and written in Babylon around 1800 BC, contains about two dozen problems of the kind we now call quadratic equations. One asks for the side of a square when the area plus the side equals three-quarters. The scribe's step-by-step method is, in effect, completing the square. Linear, quadratic and polynomial relationships have been solved for nearly four thousand years; ACT questions test whether you can choose the right form and the right method.

These questions ask you to:

- Build and interpret linear models.
- Read a **parabola** 抛物线 from standard, factored or **vertex form** 顶点式.
- Factor a **polynomial** 多项式 and use the **zero-product property** 零积性质.
- Solve polynomial inequalities and find coefficients by expanding.

Linear models

- A fixed amount plus a constant rate gives a linear model: $C = 25 + 0.40n$.
- The slope is the rate, with units; the intercept is the fixed part.
- Multiplying two variable quantities, such as two side lengths, gives a quadratic instead.

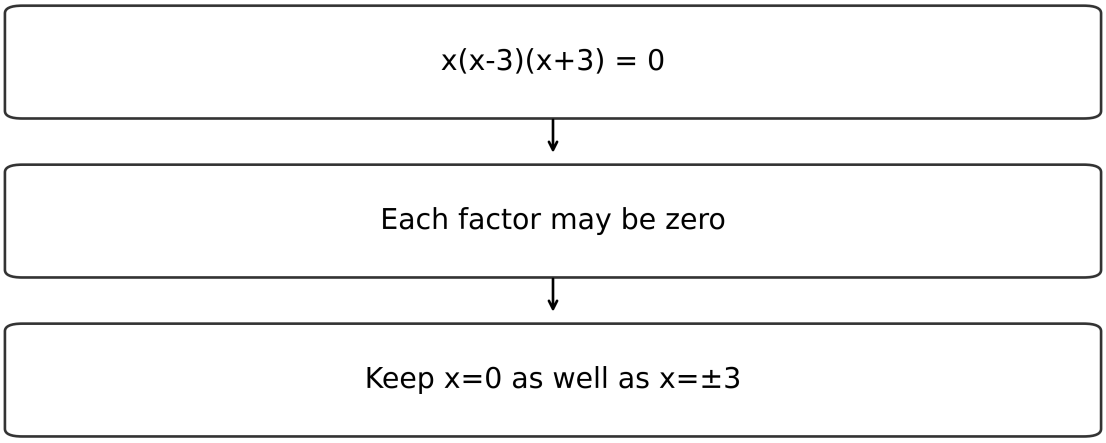
Quadratics in three forms

- Standard $ax^2 + bx + c$; factored $a(x - p)(x - q)$; vertex $a(x - h)^2 + k$.
- Complete the square to find the vertex: $x^2 - 6x = (x - 3)^2 - 9$.
- If $a > 0$ the vertex is a minimum; if $a < 0$, a maximum.

Polynomials and their zeros

- If a product is zero, at least one factor is zero.
- Factor out common factors first: $x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$.
- A repeated factor such as $(x - 1)^2$ gives a double root, where the graph touches the axis.
- Do not divide by x: it loses the root $x = 0$.

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Worked steps: x(x-3)(x+3) = 0 → Each factor may be zero → Keep x=0 as well as x=±3

Polynomial inequalities

- Find the roots, mark them on a number line, and test the sign in each interval.
- For $\geq$ or $\leq$, include the roots; for $>$ or $<$, exclude them.

Worked example: a cubic and an inequality

- $x^3 - 9x = 0$: $x(x-3)(x+3) = 0$, so $x = -3, 0, 3$.
- $(x+2)(x-5) \geq 0$: roots -2 and 5. Test $x = 0$: $(2)(-5) < 0$, so the middle is negative.
- Solution: $x \leq -2$ or $x \geq 5$, roots included.

Worked example: vertex form and a model

- $2x^2 - 12x + 23 = 2(x^2 - 6x) + 23 = 2(x-3)^2 - 18 + 23 = 2(x-3)^2 + 5$: minimum 5 at $x = 3$.
- $(2x-3)(x+4) = 2x^2 + 8x - 3x - 12 = 2x^2 + 5x - 12$: the x coefficient is 5.
- A printer charges 25 yuan plus 0.40 yuan per page. With 65 yuan: $25 + 0.40n \leq 65$, so $n \leq 100$ pages.

Common errors

- Dividing by x and losing a root.
- Getting the vertex sign wrong: $(x-3)^2$ has $h = 3$.
- Forgetting to factor out the leading coefficient before completing the square.
- Including or excluding roots wrongly in an inequality.

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- Treating a product of two lengths as linear.

Key terms

parabola: The graph of a quadratic function.
vertex form: $a(x-h)^2+k$, which shows the vertex (h,k).
polynomial: A sum of terms, each a number times a whole-number power of x.
zero-product property: If a product equals zero, at least one factor is zero.

Check yourself

- Find the roots of $(x-1)^2(x+4)=0$.
- Give the vertex of $y=(x-2)^2+3$. Maximum or minimum?
- Solve $x^2-4<0$.

Answers: $x=1$ (double) and $x=-4$. $(2,3)$, a minimum. $-2<x<2$.

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Rational equations, radical branches and systems

ACT

In 628 the Indian astronomer Brahmagupta wrote the *Brahmasphutasiddhanta*, one of the first books to give rules for calculating with zero and negative numbers. Most of his rules are the ones we use today. One is not: he stated that zero divided by zero is zero. Division by zero has no answer, and rational equations go wrong exactly there. Radical equations have a similar trap, since squaring can create false solutions. This handout is about finding every true solution and rejecting the rest.

These questions ask you to:

- Solve rational equations and state each **restriction** 限制条件 on the variable.
- Solve radical equations, checking each **candidate root** 候选根 and rejecting an **extraneous solution** 增根.
- Use the **principal square root** 算术平方根 correctly.
- Solve systems that include a curve, and describe regions of inequalities.

Rational equations

- Before anything else, list the values that make a denominator zero.
- Multiply every term by the lowest common denominator, then solve.
- Reject any answer on your list. An equation can end with no solution.
- When simplifying a rational expression, the original restrictions remain.

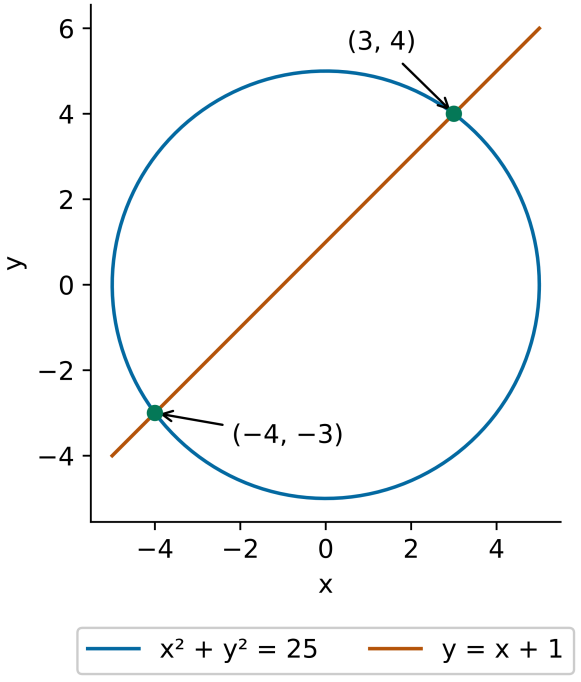
Radical equations

- $\sqrt{x}$ means the principal (non-negative) root, so $\sqrt{A} = x$ requires $x \geq 0$.
- Isolate the root, square both sides, solve, and check every candidate in the original equation.
- $x^2 = 9$ has two branches, $x = 3$ or $x = -3$; but $\sqrt{9}$ is only 3.

Systems with a curve

- Substitute the linear equation into the curved one to get a single equation, often a quadratic.
- Find the second coordinate from the linear equation.
- The number of real solutions of that quadratic is the number of intersection points.

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Substituting the line into the circle gives both crossing points

For the figure: $x^2 + (x+1)^2 = 25$ gives $x^2 + x - 12 = 0$, so $x = 3$ or $x = -4$.

Systems of inequalities

- Draw each boundary: solid for $\leq$ or $\geq$, dashed for $<$ or $>$.
- The solution is the region satisfying all the inequalities; test a point to check.

Worked example: a circle and a line

Solve $x^2 + y^2 = 25$ and $y = x - 1$.

- Substitute: $x^2 + (x-1)^2 = 25$, so $2x^2 - 2x - 24 = 0$, that is $x^2 - x - 12 = 0$.
- $(x-4)(x+3) = 0$, so $x = 4$ or $x = -3$.
- Points: $(4, 3)$ and $(-3, -4)$. Check: $16 + 9 = 25$ and $9 + 16 = 25$.

Worked example: rejecting candidates

- $\sqrt{3x+4} = x$: square to get $x^2 - 3x - 4 = (x-4)(x+1) = 0$. Check: $\sqrt{16} = 4$ works; $\sqrt{1} = 1 \neq -1$, so reject -1. Answer: $x = 4$.
- $\dfrac{x}{x-2} = \dfrac{2}{x-2} + 3$, with $x \neq 2$: multiply by $x - 2$ to get $x = 2 + 3(x-2)$, so $x = 2$.
- But $x = 2$ is excluded, so this equation has no solution.

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Common errors

- Not listing the restrictions before solving.
- Keeping an answer that makes a denominator zero.
- Forgetting to check candidates after squaring.
- Writing $\sqrt{9} = \pm 3$.
- Giving only x-values when the question asks for points.

Key terms

restriction: A value the variable cannot take, such as one that makes a denominator zero.

candidate root: A possible solution that must be checked.

extraneous solution: A candidate that comes from the method but does not satisfy the original equation.

principal square root: The non-negative square root, written $\sqrt{x}$.

Check yourself

- Solve $\dfrac{1}{x} + \dfrac{1}{2} = 1$.
- Solve $\sqrt{x+6} = x$.
- How many points do $y = x^2$ and $y = -1$ share?

Answers: $x = 2$. $x = 3$ ($x = -2$ is extraneous). None.

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Functions, composition, transformations and inverses

ACT

Functions: Composition, Transformations and Inverses

In 1734 Leonhard Euler, working in St Petersburg, wrote $f\left(\frac{x}{a}+c\right)$ in a paper: one of the first uses of the notation $f(x)$ for a function. The idea he was naming, a rule that gives exactly one output for each input, became central to all of mathematics. The notation lets you write chains of rules, such as $f(g(x))$, and ask whether a rule can be undone. ACT function questions test exactly these: notation, composition, transformation and inverses.

These questions ask you to:

- Find the **domain** 定义域 and **range** 值域 of a function.
- Form a **composition** 函数复合 and find its domain.
- Describe a **transformation** 变换 of a graph.
- Find an **inverse function** 反函数, which needs a **one-to-one** 单射 function.

Notation, domain and range

- $f(3)$ means the output when the input is 3.
- The domain excludes inputs that divide by zero or take an even root of a negative number.
- The range is the set of outputs; a graph or a vertex often shows it.

Composition

- $f(g(x))$ means: apply g, then apply f to the result.
- The domain of $f(g(x))$ contains only inputs where g is defined and $g(x)$ is allowed in f.
- Usually $f(g(x)) \neq g(f(x))$.

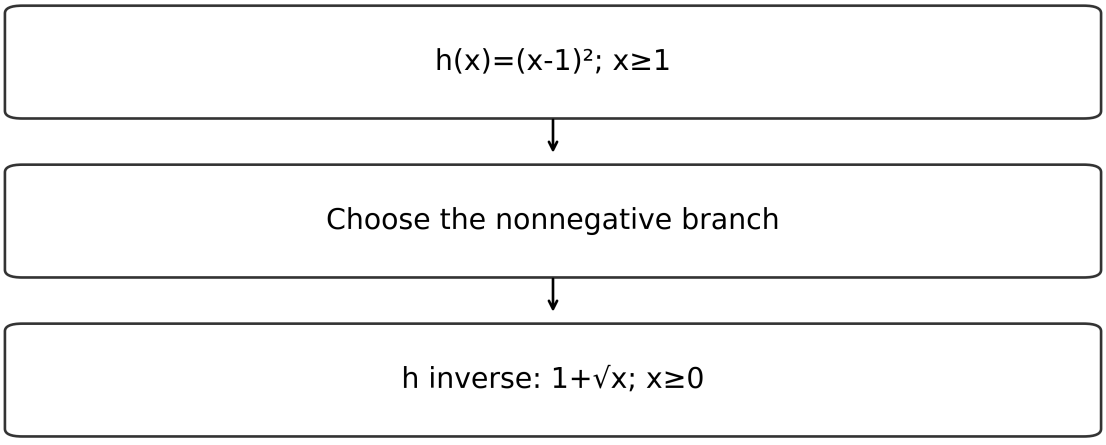
Transformations

- $f(x)+k$: up k. $f(x-h)$: right h. $af(x)$: vertical stretch by a, reflected if $a<0$. $f(-x)$: reflection in the y-axis.
- Track a point: if $(1,1)$ is on $y=x^2$, then on $y=3(x+4)^2-2$ it moves to $(-3,1)$.

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Inverse functions

- An inverse undoes a function: if $f(a) = b$, then $f^{-1}(b) = a$.
- Only a one-to-one function, which passes the horizontal-line test, has an inverse. Otherwise, restrict the domain first.
- To find it: write $y = f(x)$, solve for x, then swap the letters. The domain and range swap too.



Worked steps: h(x)=(x-1)²; x≥1 → Choose the nonnegative branch → h inverse: 1+√x; x≥0

Worked example: composition both ways

Let $f(u) = \dfrac{1}{u-3}$ and $g(x) = x^2 + 3$.

- $f(g(x)) = \dfrac{1}{x^2+3-3} = \dfrac{1}{x^2}$, with $x \neq 0$.
- $g(f(4))$: $f(4) = 1$, then $g(1) = 4$.
- $f(g(4)) = \dfrac{1}{16}$: a different answer, because the order changed.

Worked example: an inverse on a branch

Restrict $h(x) = (x+2)^2$ to $x \leq -2$.

- On this domain $x + 2 \leq 0$, so solving $y = (x+2)^2$ gives $x + 2 = -\sqrt{y}$.
- $h^{-1}(x) = -2 - \sqrt{x}$, with domain $x \geq 0$ and range $y \leq -2$.
- The + branch, $-2 + \sqrt{x}$, gives values above -2, outside the original domain.

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Common errors

- Applying f before g in $f(g(x))$.
- Ignoring the domain of the inner function.
- Shifting $f(x+4)$ to the right instead of the left.
- Finding an inverse for a function that is not one-to-one.
- Choosing the wrong square-root branch.

Key terms

domain: The set of allowed inputs.

range: The set of possible outputs.

composition: Applying one function to the output of another.

transformation: A shift, stretch or reflection of a graph.

inverse function: The function that reverses another.

one-to-one: Giving a different output for each different input.

Check yourself

- Find the inverse of $f(x) = 2x - 6$.
- Is $f(x) = x^2$, on all real numbers, one-to-one?
- Write $y = \sqrt{x}$ shifted 3 left and 1 up.

Answers: $f^{-1}(x) = \dfrac{x+6}{2}$. No: $f(2) = f(-2)$. $y = \sqrt{x+3} + 1$.

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Sequences, finite sums, exponentials and logarithms

ACT

In 1614 the Scottish landowner John Napier published *Mirifici Logarithmorum Canonis Descriptio*, a book of tables that turned multiplication into addition. Astronomers who spent days on hand calculations could now look up two numbers, add them, and look up the answer. The London professor Henry Briggs travelled to Edinburgh to meet Napier, and the two agreed on base-10 logarithms, which Briggs published in 1617. Logarithms are the inverse of exponentials, and both are linked to sequences that grow by adding or by multiplying.

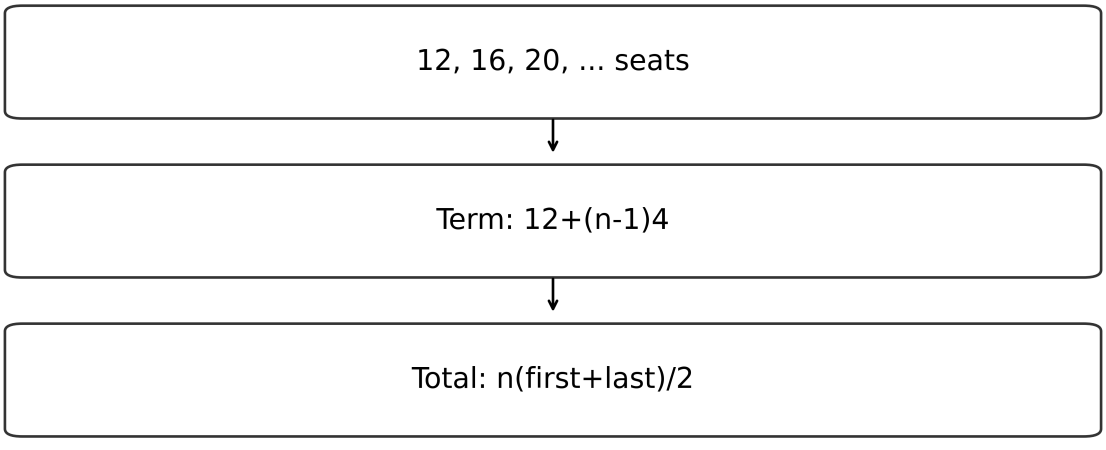
These questions ask you to:

- Find terms and sums of an **arithmetic sequence** 等差数列.
- Find terms and sums of a **geometric sequence** 等比数列 with a **common ratio** 公比.
- Write exponential growth and decay models.
- Use a **logarithm** 对数, and keep its **logarithm argument** 真数 positive.

Arithmetic sequences

$$a_n = a_1 + (n-1)d \qquad S_n = \frac{n(a_1 + a_n)}{2}$$

- Each term adds the same difference d.
- The sum is the number of terms times the average of the first and last.



Worked steps: 12, 16, 20, ... seats → Term: 12+(n-1)4 → Total: n(first+last)/2

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Geometric sequences

$$a_n = a_1 r^{n-1} \qquad S_n = \frac{a_1(1-r^n)}{1-r},\ r \neq 1$$

- Each term multiplies by the same ratio r.
- If $r = 1$, all terms are equal and $S_n = na_1$.

Exponential models

- $N(t) = N_0 b^{t/T}$: start N_0, factor b for each period T.
- Growth of $r\%$ per period: $b = 1 + r/100$; decay: $b = 1 - r/100$.

Logarithms

- $\log_b y = x$ means $b^x = y$. The argument y must be positive.
- $\log(MN) = \log M + \log N$; $\log(M/N) = \log M - \log N$; $\log(M^k) = k \log M$.
- There is no rule for $\log(M + N)$.
- To solve $b^x = c$, take logs: $x = \dfrac{\log c}{\log b}$.

Worked example: seats in rows

Rows have 12, 16, 20, … seats.

- $d = 4$, so row 8 has $a_8 = 12 + 7(4) = 40$ seats.
- Total in 8 rows: $S_8 = \dfrac{8(12 + 40)}{2} = 208$ seats.
- This assumes every row adds exactly 4 seats.

Worked example: geometric terms and logarithms

- For 5, 15, 45, …: $a_5 = 5(3)^4 = 405$, and $S_5 = \dfrac{5(1-3^5)}{1-3} = \dfrac{5(-242)}{-2} = 605$.
- $\log_2(x-3) = 5$: need $x > 3$; then $x - 3 = 2^5 = 32$, so $x = 35$.
- $3^x = 20$: $x = \dfrac{\log 20}{\log 3} \approx \dfrac{1.301}{0.477} \approx 2.73$.

Common errors

- Using n instead of $n - 1$ in the term formula.
- Using the geometric sum formula when $r = 1$.
- Splitting $\log(x + 3)$ into $\log x + \log 3$.
- Forgetting that the logarithm's argument must be positive.
- Mixing up the difference and the ratio.

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Key terms

arithmetic sequence: A sequence with a constant difference between terms.
geometric sequence: A sequence with a constant ratio between terms.
common ratio: The number each term of a geometric sequence is multiplied by.
logarithm: The power to which a base must be raised to give a number.
logarithm argument: The number whose logarithm is taken; it must be positive.

Check yourself

- Find the 7th term of 3, 7, 11, ….
- Evaluate $\log_3 81$.
- Find $2 + 4 + 8 + \cdots + 64$.

Answers: $3 + 6(4) = 27$. 4. $2(2^6 - 1) = 126$.

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Trigonometric graphs, identities and general triangles

ACT

In 1595 the German mathematician Bartholomäus Pitiscus, chaplain to a prince in Heidelberg, published a book called *Trigonometria*, from the Greek for "triangle measurement". The word stuck. The ideas were much older: astronomers from Hipparchus in the second century BC to scholars in India and the Islamic world had built tables of chords and sines to track the planets. Today trigonometry measures triangles that are not right-angled, and describes anything that repeats, from sound waves to tides. ACT questions cover both.

These questions ask you to:

- Use sine, cosine and tangent in right triangles and on the unit circle.
- Read the **amplitude** 振幅, **period** 周期 and **midline** 中线 of a sine or cosine graph.
- Use the **sine rule** 正弦定理 and the **cosine rule** 余弦定理 in any triangle.
- Use the **included angle** 夹角 to find a triangle's area, and basic identities.

Right triangles and the unit circle

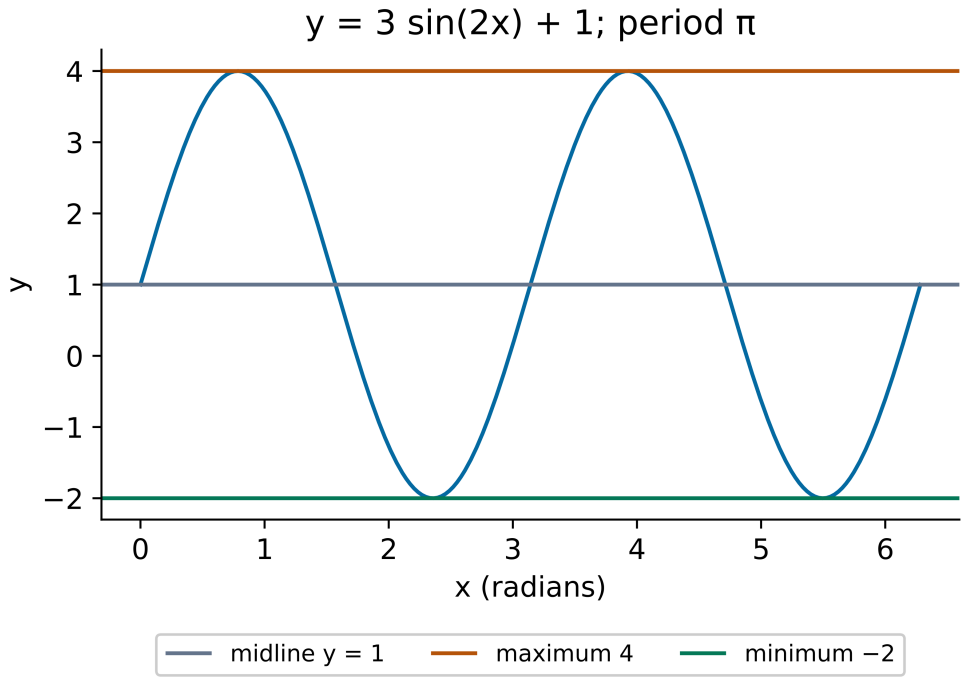
- $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$.
- On the unit circle, the point at angle θ is $(\cos\theta, \sin\theta)$, so $\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.
- Sine is positive in quadrants I and II; cosine in I and IV. So $\sin\theta = \frac{1}{2}$ has two answers in a full turn: $30°$ and $150°$.

Graphs of sine and cosine

$$y = a\sin(bx) + d$$

- Amplitude $|a|$: half the distance from maximum to minimum. A negative a reflects the wave; the amplitude is still positive.
- Period $\dfrac{2\pi}{b}$ (in radians) or $\dfrac{360°}{b}$.
- Midline $y = d$; range from $d - |a|$ to $d + |a|$.

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Amplitude 3 about the midline $y = 1$; the factor 2 makes the period π

Triangles that are not right-angled

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad c^2 = a^2 + b^2 - 2ab\cos C \qquad \text{Area} = \tfrac{1}{2}ab\sin C$$

- Use the sine rule when you know a side and its opposite angle.
- Use the cosine rule with two sides and the angle between them, or with three sides.
- The area formula needs the angle between the two given sides.

Worked example: a graph

For $y = -4\sin(2x) + 3$, with x in radians:

- Amplitude 4 (the minus sign reflects the wave); period $\dfrac{2\pi}{2} = \pi$; midline $y = 3$.
- Range: $3 - 4 = -1$ to $3 + 4 = 7$.
- At $x = \frac{\pi}{4}$: $y = -4\sin\frac{\pi}{2} + 3 = -4 + 3 = -1$, a minimum.

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Worked example: two general triangles

A triangle has sides 6 cm and 10 cm with $60°$ between them.

- $c^2 = 36 + 100 - 2(6)(10)\cos 60° = 136 - 60 = 76$, so $c = 2\sqrt{19} \approx 8.72$ cm.
- Area $= \frac{1}{2}(6)(10)\sin 60° = 30 \cdot \frac{\sqrt{3}}{2} = 15\sqrt{3} \approx 26.0$ cm^2.
- Another triangle has 8 cm opposite $30°$ and an angle of $45°$. Sine rule: $b = \dfrac{8\sin 45°}{\sin 30°} = \dfrac{8(\sqrt{2}/2)}{1/2} = 8\sqrt{2}$ cm.

Common errors

- Using SOH-CAH-TOA in a triangle that is not right-angled.
- Giving only the acute solution of $\sin\theta = \frac{1}{2}$.
- Writing a negative amplitude.
- Using the period $2\pi b$ instead of $\dfrac{2\pi}{b}$.
- Using a side angle in the area formula instead of the included angle.

Key terms

amplitude: Half the distance between the maximum and minimum of a wave.

period: The length of one full cycle.

midline: The horizontal line halfway between maximum and minimum.

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$.

included angle: The angle between two given sides.

Check yourself

- Find every θ with $0° \le \theta < 360°$ and $\sin\theta = \frac{1}{2}$.
- What is the period of $y = \cos(3x)$, in radians?
- A right triangle has legs 9 and 12. Find sin of the angle opposite 9.

Answers: $30°$ and $150°$. $\frac{2\pi}{3}$. The hypotenuse is 15, so $\frac{9}{15} = \frac{3}{5}$.

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Plane transformations, coordinate reasoning and proof

ACT

In 1872 the 23-year-old Felix Klein, newly appointed professor at Erlangen in Germany, set out a programme that changed how geometry was understood. Each kind of geometry, he proposed, is the study of the properties that stay the same under a group of transformations. Lengths and angles survive slides, turns and flips; shapes but not sizes survive enlargements. The ACT asks the same questions on a coordinate grid: what does a transformation do, and what does it preserve?

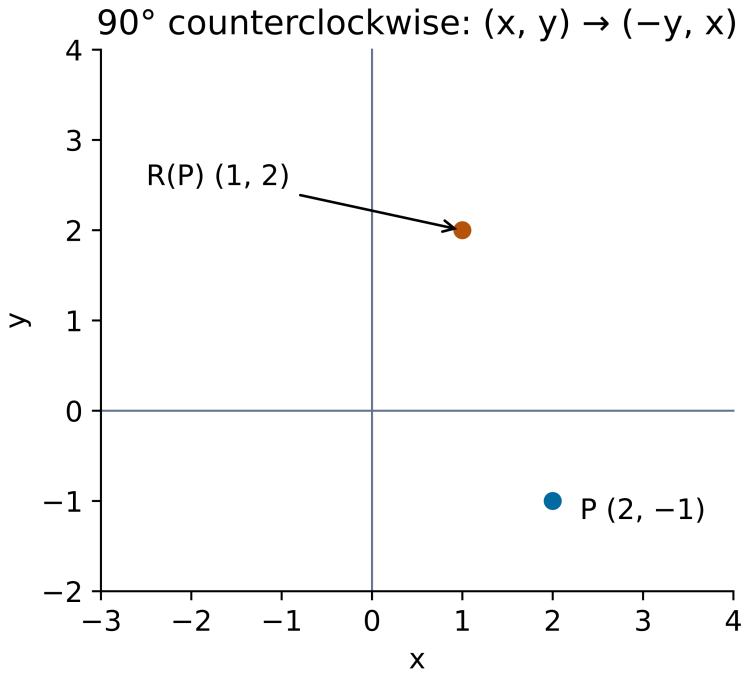
These questions ask you to:

- Apply each of the **rigid motions** 刚体变换: **translation** 平移, reflection and rotation.
- Use a **dilation** 位似 and **similarity** 相似, and decide on **congruence** 全等.
- Use distance, midpoint and slope, including the **perpendicular bisector** 垂直平分线.
- Give a reason that holds for every case, not one measured diagram.

Rigid motions

- Translation by (a, b): $(x, y) \to (x + a, y + b)$.
- Reflection in the x-axis: $(x, -y)$; in the y-axis: $(-x, y)$; in $y = x$: (y, x).
- Rotation about the origin by 90° anticlockwise: $(-y, x)$; by 180°: $(-x, -y)$.
- Rigid motions keep lengths and angles. The order of two motions can change the result.

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A 90° counterclockwise rotation sends (x, y) to (−y, x)

Dilations, similarity and congruence

- A dilation with scale factor k about the origin: (kx, ky). Lengths $\times k$, areas $\times k^2$, angles unchanged.
- Figures are congruent if a rigid motion maps one onto the other; similar if a rigid motion and a dilation do.
- Triangles with equal angles are similar; SSS, SAS, ASA and AAS prove congruence.

Coordinate tools

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Parallel lines have equal slopes; perpendicular slopes multiply to -1.
- The perpendicular bisector of AB passes through the midpoint with the negative reciprocal slope. Every point on it is the same distance from A and B.

Reasons and proof

- A proof gives a reason that works for every case: “equal radii”, “corresponding angles on parallel lines”.
- Measuring one diagram does not prove a general result.

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- In a construction, say why each step works, for example why arcs of equal radius find points equidistant from A and B.

Worked example: order matters

Rotate $A(2, -3)$ by $90°$ anticlockwise about the origin, then translate by $(4, 1)$.

- Rotate: $(-y, x) = (3, 2)$. Translate: $(7, 3)$.
- Other order: translate to $(6, -2)$, then rotate to $(2, 6)$.
- The results differ, so the order of the transformations matters.

Worked example: a segment and two triangles

$A = (-1, 2)$ and $B = (5, 10)$.

- Length: $\sqrt{6^2 + 8^2} = 10$. Midpoint: $(2, 6)$.
- Slope of AB: $\frac{8}{6} = \frac{4}{3}$, so the perpendicular bisector has slope $-\frac{3}{4}$: $y - 6 = -\frac{3}{4}(x - 2)$.
- Two triangles with angles $40°$, $60°$, $80°$ and longest sides 9 cm and 15 cm are similar, not congruent. Length ratio $\frac{15}{9} = \frac{5}{3}$; area ratio $\frac{25}{9}$.

Common errors

- Using $(y, -x)$ for an anticlockwise rotation (that is clockwise).
- Assuming two transformations can be done in either order.
- Calling triangles congruent because their angles are equal.
- Using the same slope, not the negative reciprocal, for a perpendicular line.
- Treating a measurement on one diagram as a proof.

Key terms

rigid motions: Transformations that keep lengths and angles: translations, reflections and rotations.

translation: A slide of every point by the same vector.

dilation: An enlargement or reduction by a scale factor about a centre.

similarity: Same shape, with lengths in a fixed ratio.

congruence: Same shape and same size.

perpendicular bisector: The line through a segment's midpoint at right angles to it.

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Check yourself

- Reflect $(3, -4)$ in the x-axis.
- Lines have slopes 2 and $-\frac{1}{2}$. Are they parallel, perpendicular or neither?
- Find the midpoint and length of the segment from $(0, 0)$ to $(8, 6)$.

Answers: $(3, 4)$. Perpendicular, since $2 \times (-\frac{1}{2}) = -1$. $(4, 3)$; 10.

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Conics, circle relationships and geometric measurement

ACT

Around 200 BC Apollonius of Perga wrote eight books on the *Conics*: the curves you get by slicing a cone with a plane. He gave three of them the names we still use: ellipse, parabola and hyperbola. Eighteen centuries later, in 1609, Johannes Kepler showed that Mars moves round the Sun not in a circle but in an ellipse. Conic sections describe orbits, satellite dishes and bridges; on the ACT you read their equations and use the measurement facts that go with circles and solids.

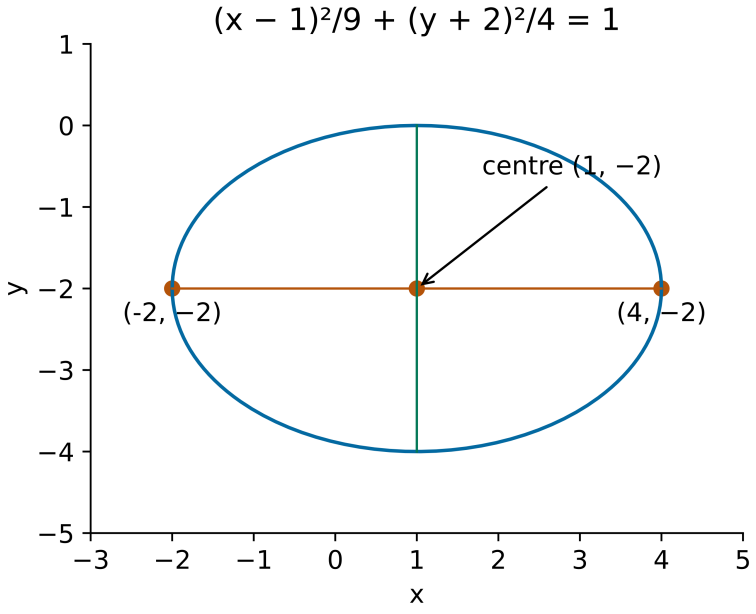
These questions ask you to:

- Read the centre and **semiaxes** 半轴 of an **ellipse** 椭圆.
- Recognise a **hyperbola** 双曲线 and find the vertex and **focus** 焦点 of a **parabola** 抛物线.
- Use lengths of a **tangent** 切线 and a **secant** 割线 drawn from one point.
- Find volumes and surface areas, and how they scale.

Circles and ellipses

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

- Centre (h,k); semiaxis a across, b up and down. If $a=b$, it is a circle of radius a.
- The ends of the axes are $(h\pm a,k)$ and $(h,k\pm b)$.



The denominators 9 and 4 give semi-axes 3 across and 2 up and down

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Parabolas and hyperbolas

- $y^2 = 4px$: vertex at the origin, focus $(p, 0)$; it opens right if $p > 0$, left if $p < 0$. Likewise $x^2 = 4py$ has focus $(0, p)$.
- A minus sign between the squared terms, as in $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, gives a hyperbola.
- Classify by the signs: both squared terms positive (ellipse or circle), opposite signs (hyperbola), only one squared term (parabola).

Tangents, secants and chords

- From an outside point, a tangent of length t and a secant with outside part e and whole length w satisfy $t^2 = e\,w$.
- Two secants from the same point: $e_1 w_1 = e_2 w_2$.
- Two chords crossing inside a circle: the products of their parts are equal.
- A tangent is perpendicular to the radius at the point of contact.

Measurement and scaling

- Cylinder: $V = \pi r^2 h$, surface area $2\pi r^2 + 2\pi r h$. Cone: $V = \frac{1}{3}\pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$, area $4\pi r^2$.
- If every length is multiplied by k, areas are multiplied by k^2 and volumes by k^3.

Worked example: reading equations

- $\dfrac{(x+2)^2}{25} + \dfrac{(y-1)^2}{9} = 1$: centre $(-2, 1)$, semiaxes 5 and 3.
- Axis ends: $(-7, 1)$, $(3, 1)$, $(-2, -2)$, $(-2, 4)$.
- $y^2 = -12x$: $4p = -12$, so $p = -3$. Vertex $(0, 0)$, focus $(-3, 0)$; it opens to the left.

Worked example: a secant and a cylinder

- A tangent from a point has length 6 cm; a secant from it has outside part 4 cm. Then $w = \dfrac{6^2}{4} = 9$ cm, and the part inside the circle is $9 - 4 = 5$ cm.
- A cylinder with $r = 3$ cm and $h = 8$ cm: $V = \pi(9)(8) = 72\pi$ cm^3; area $= 18\pi + 48\pi = 66\pi$ cm^2.
- Doubling every length: volume $\times 8 = 576\pi$ cm^3; area $\times 4 = 264\pi$ cm^2.

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Common errors

- Reading a^2 as the semiaxis instead of a.
- Getting the sign of the centre wrong.
- Using the inside part of a secant instead of its whole length.
- Mixing up p and $4p$ for a parabola.
- Multiplying volume by k instead of k^3.

Key terms

ellipse: An oval curve with equation $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$.

semiaxes: The distances a and b from the centre to the ends of an ellipse's axes.

hyperbola: A conic with two separate branches, with a minus sign between the squared terms.

parabola: A conic with only one squared variable.

focus: A fixed point used to define a conic.

tangent: A line touching a circle at exactly one point.

secant: A line that cuts a circle at two points.

Check yourself

- Classify $\dfrac{x^2}{16}-\dfrac{y^2}{9}=1$ and $x^2+y^2=16$.
- Find the circumference and area of the circle $x^2+y^2=16$.
- Two secants from one point: the first has outside part 3 and whole length 12; the second has outside part 4. Find its whole length.

Answers: hyperbola; circle. 8π and 16π. $\frac{3\times12}{4}=9$.

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Data displays, normal distributions and association

ACT

In 1733 the French mathematician Abraham de Moivre, living in London and earning his money partly by solving problems for gamblers, printed a short paper in Latin. He showed that when a coin is tossed many times, the chances of the different numbers of heads follow a smooth bell-shaped curve. Later scientists, including Gauss and the Belgian Adolphe Quetelet, found the same curve in measurement errors and in human heights. It is now called the normal distribution. ACT data questions ask you to choose and read displays, use that bell curve, and judge what an association shows.

These questions ask you to:

- Read and choose a data display: histogram, box plot, bar chart or scatterplot.
- Find the **median** 中位数, quartiles and **interquartile range** 四分位距, using a stated **quartile convention** 四分位数约定.
- Use a **normal distribution** 正态分布 and its **standard deviation** 标准差.
- Read a **frequency** 频数 table, and use a fitted line and its **residual** 残差 without claiming cause.

Choosing a display

- Histogram: one quantitative variable grouped into intervals.
- Box plot: the five-number summary (minimum, Q_1, median, Q_3, maximum).
- Bar chart: counts in categories. Scatterplot: pairs of quantitative values.
- From a histogram you can find proportions, but not the exact mean, since values within a bin are unknown.

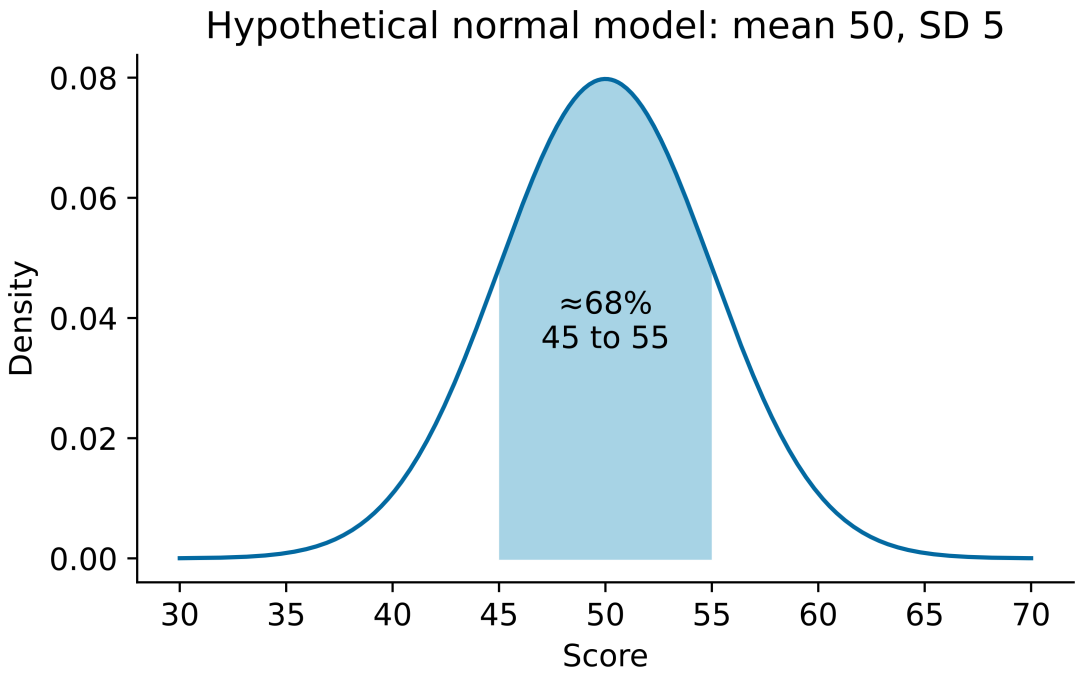
Quartiles and spread

- Order the data. The median is the middle value.
- One common convention: Q_1 is the median of the lower half and Q_3 of the upper half. The ACT states its convention when it matters.
- IQR $= Q_3 - Q_1$; it is not affected by extreme values.

The normal model

- Bell-shaped and symmetric about its mean μ, with spread σ.
- About 68% lies within 1σ of the mean, 95% within 2σ, 99.7% within 3σ.
- These are model proportions; a small real sample may not match them exactly.
- If every value becomes $ax + b$ (with $a > 0$), the mean becomes $a\mu + b$ and the standard deviation $a\sigma$.

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About 68% of a normal model lies within one standard deviation of the mean

Association and fitted lines

- Describe a scatterplot by direction, form and strength, and note any outliers.
- A fitted line summarises the trend; it does not join every point.
- Residual = observed − predicted.
- Association is not causation: another factor may explain both variables.

Worked example: a normal model

Scores follow a normal model with mean 80 and standard deviation 6.

- About 68% lie between $80 - 6 = 74$ and 86; about 95% between 68 and 92.
- So about 2.5% score above 92.
- If each score becomes $2x+5$: mean $2(80)+5 = 165$; standard deviation $2(6) = 12$.

Worked example: a five-number summary

Data: 2, 4, 4, 6, 9, 11, 12, 16.

- Median $= \dfrac{6+9}{2} = 7.5$.
- Lower half 2, 4, 4, 6: $Q_1 = 4$. Upper half 9, 11, 12, 16: $Q_3 = 11.5$.
- Summary $(2, 4, 7.5, 11.5, 16)$; IQR $= 7.5$. A box plot shows exactly these values.
- A fitted line $y = 4x + 3$ predicts 23 at $x = 5$; if the observed value is 20, the residual is -3.

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Common errors

- Choosing a bar chart for quantitative data.
- Calculating an “exact” mean from grouped data.
- Adding the constant b to the standard deviation.
- Treating 68% as exact for a small sample.
- Reading a positive association as proof of cause.

Key terms

median: The middle value of ordered data.

interquartile range: $Q_3 - Q_1$, the spread of the middle half.

quartile convention: The stated rule for finding quartiles.

normal distribution: A symmetric, bell-shaped model of data.

standard deviation: A measure of typical distance from the mean.

frequency: The number of observations in a category or interval.

residual: Observed value minus predicted value.

Check yourself

- In a normal model with mean 100 and standard deviation 15, about what percentage is above 130?
- A histogram has bins [0, 10), [10, 20), [20, 30] with frequencies 2, 6, 2. What proportion is below 20?
- Adding 5 to every value: what happens to the standard deviation?

Answers: about 2.5%. $\frac{8}{10} = 80\%$. It does not change.

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Sampling inference, expected value and counting

ACT

In 1657 the Dutch scientist Christiaan Huygens published *De Ratiociniis in Ludo Aleae* ("On Reasoning in Games of Chance"), the first printed book on probability. He had heard about the Pascal–Fermat letters while visiting Paris, and he worked out his own answers. His key idea was the value of a chance: what a game is worth on average if played many times. We now call it expected value. ACT questions combine it with counting, probability without replacement and the logic of samples.

These questions ask you to:

- Count arrangements, telling a **permutation** 排列 from a **combination** 组合.
- Find probabilities, including **conditional probability** 条件概率 and draws without replacement.
- Calculate an **expected value** 期望值.
- Interpret a survey's **margin of error** 误差范围 and what a study can claim.

Counting

- Multiplication principle: if one choice has m options and the next n, together there are mn.
- Order matters (a permutation): ${}_nP_r = n(n-1)\cdots(n-r+1)$.
- Order does not matter (a combination): ${}_nC_r = \dfrac{{}_nP_r}{r!}$, since each group was counted $r!$ times.

Choose 2 of 4: six unordered pairs; AB and BA count once

AB	AC	AD
BC	BD	CD

Choosing 2 of 4 gives six pairs, because AB and BA are the same pair

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Probability without replacement

- Without replacement, the second draw depends on the first: the totals change.
- $P(A \text{ and } B) = P(A) \times P(B \mid A)$.
- "Exactly one" can happen in more than one order: add the cases.

Expected value

$$E(X) = \sum x\,P(x)$$

- Multiply each outcome by its probability and add.
- Net gain = expected payout − cost.
- The expected value is a long-run average; a single play may never give that amount.

Samples and studies

- An estimate with a margin of error gives an interval of plausible values for the population that was sampled.
- A large sample of volunteers can still be biased; random selection reduces bias.
- Random assignment to treatments supports a causal claim; random sampling supports generalising.

Worked example: roles, committees and draws

- Six students choose a chair and a different secretary: $6 \times 5 = 30$ ways (order matters).
- An unranked pair of two: $\dfrac{30}{2} = 15$, since each pair was counted twice.
- A bag has 3 red and 2 blue. $P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$.
- $P(\text{exactly one red}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{3}{5}$.

Worked example: a game and a survey

A game pays 12 tokens with probability 0.1, 3 tokens with probability 0.4, and nothing otherwise. Entry costs 2 tokens.

- $E(\text{payout}) = 12(0.1) + 3(0.4) + 0(0.5) = 2.4$ tokens; expected net gain = 0.4 token per play.
- The actual net results are 10, 1 or -2: never 0.4.
- A random sample of 500 library members finds 62% support, margin 4 points: the interval is 58% to 66% of members, not of all town residents.

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Common errors

- Using a combination when order matters, or the reverse.
- Keeping the same denominator for a second draw without replacement.
- Counting only one order for "exactly one".
- Expecting the expected value on a single play.
- Generalising a survey beyond the sampled population.

Key terms

permutation: An arrangement in which order matters.

combination: A selection in which order does not matter.

conditional probability: The probability of an event given that another has happened.

expected value: The long-run average outcome, $\sum x\,P(x)$.

margin of error: The amount added to and subtracted from an estimate to allow for sampling variation.

Check yourself

- In how many orders can 4 books stand on a shelf?
- How many ways can you choose 2 of 5 people (unordered)?
- What is the expected value of one roll of a fair six-sided die?

Answers: $4 \times 3 \times 2 \times 1 = 24$. $\frac{5\times4}{2} = 10$. $\frac{1+2+3+4+5+6}{6} = 3.5$.

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Essential skills in a multi-step practical model

ACT

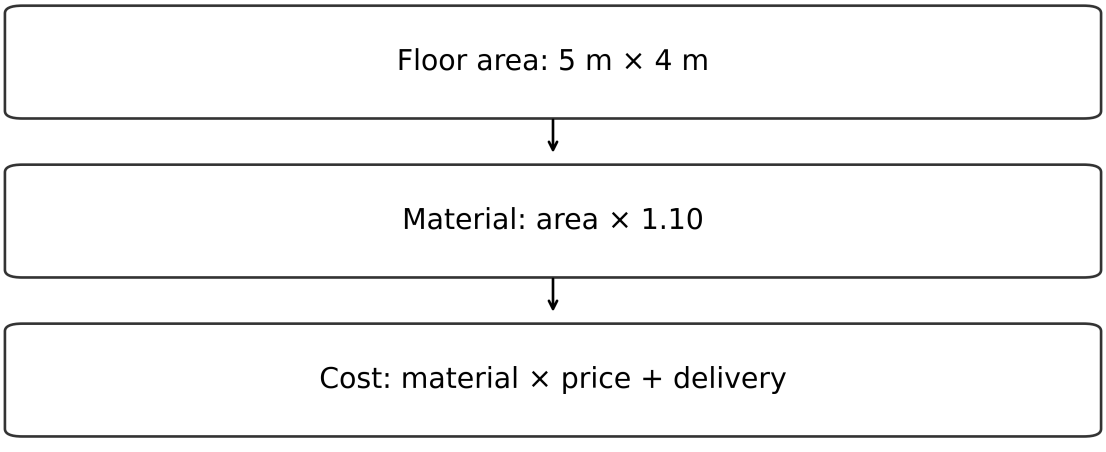
At 5:29 in the morning on 16 July 1945, the first atomic bomb was tested in the New Mexico desert. About 40 seconds after the flash, when the blast wave reached him some 16 kilometres away, the physicist Enrico Fermi dropped small pieces of paper from his hand and watched how far they were blown. From that distance and a quick calculation, he estimated the explosion's energy as equal to about 10,000 tonnes of TNT. Careful instrument measurements later gave about 21,000 tonnes. His estimate was within a factor of about two. ACT practical problems need the same habits: break a situation into steps, carry units, and check whether the answer is reasonable.

These questions ask you to:

- Turn a real situation into a model, stating each **model assumption** 模型假设.
- Use area, percentages, a **unit rate** 单位率 and unit conversions in sequence.
- Read a **scale drawing** 比例图.
- Check a model against data using a **residual** 残差, and make a quick **estimate** 估算 to test reasonableness.

Planning the model

- List the quantities you know, with units, and the one you need.
- Write the steps in order before calculating: area, then material, then cost.
- State assumptions, such as "wastage is 10% of the area" or "the price per unit is constant".



Worked steps: Floor area: 5 m × 4 m → Material: area × 1.10 → Cost: material × price + delivery

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Rates, percentages and units

- A percentage allowance multiplies: 10% extra means $\times 1.10$, applied to the quantity named (area, not each side).
- Cost = unit price $\times$ quantity + fixed charges.
- Convert units before combining quantities; write units at each step.

Scale drawings

- A scale of $1 : 50$ means 1 cm on the plan is 50 cm in reality.
- Lengths divide by the scale; areas divide by the scale squared.

Checking and revising a model

- Compare predictions with observations: residual = observed − predicted.
- A constant residual suggests the fixed part (intercept) is wrong; a residual that grows with the input suggests the rate (slope) is wrong.
- A rough estimate in round numbers catches errors in the calculation.

Worked example: flooring a room

A room is 5 m by 4 m. Allow 10% extra flooring for wastage. Flooring costs 35 yuan per m^2, and delivery is 60 yuan.

- Area: $5 \times 4 = 20$ m^2. Material: $1.10 \times 20 = 22$ m^2.
- Cost: $35 \times 22 + 60 = 770 + 60 = 830$ yuan.
- Estimate: about $20 \times 35 = 700$, plus extras: 830 is reasonable.
- On a $1 : 50$ plan: $500 \div 50 = 10$ cm by $400 \div 50 = 8$ cm.

Worked example: revising a delivery model

A model $C = 6 + 4d$ yuan is 5 yuan below the observed bill at 2, 4 and 6 km.

- Every residual is +5, the same at each distance.
- So raise the fixed part: $C = 11 + 4d$.
- If instead the residuals grew with distance (say +2, +4, +6), the per-kilometre rate would be too low. Either way, check the actual fee rules before trusting the revision.

Common errors

- Applying a 10% allowance to each side length instead of the area.
- Forgetting a fixed charge.
- Dividing an area by the scale instead of the scale squared.
- Changing the slope when the residuals show a constant gap.

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- Skipping the reasonableness check.

Key terms

model assumption: A condition a model takes to be true, such as a constant price.
unit rate: The amount for one unit, such as yuan per square metre.
scale drawing: A drawing in which every length is reduced by the same factor.
residual: Observed value minus the model's prediction.
estimate: A quick approximate value used to check an answer.

Check yourself

- A wall is 2 m by 3 m, and 1 L of paint covers 10 m^2. How much paint is needed for one coat?
- On a $1:200$ plan, a wall is 3 cm long. How long is it really?
- A model's residuals are -1, -3, -5 as the input rises. Which part of the model should you check?

Answers: 0.6 L. 6 m. The rate (slope): the gap grows with the input.

English: purpose, focus and argumentative support

ACT

ACT English: Purpose, Focus and Argumentative Support

When Charles Darwin published *On the Origin of Species* in November 1859, he called the book "one long argument". Every chapter served a single claim, and he did not avoid the objections: one chapter, "Difficulties on Theory", raised the strongest problems himself, such as how something as complex as the eye could evolve, and answered them. Readers who disagreed with him still had to admit that the book stayed on its point. ACT English asks you to judge writing in the same way: does each sentence serve the paragraph's purpose, and does the support really support it?

These questions ask you to:

- Name a paragraph's purpose and test the **relevance** 相关性 of each sentence and **detail** 细节.
- Judge whether **evidence** 证据 supports the claim it is attached to.
- Decide whether a **counterargument** 反论点 helps the writer's **argument** 论证.
- Choose a qualified conclusion that the passage's facts allow.

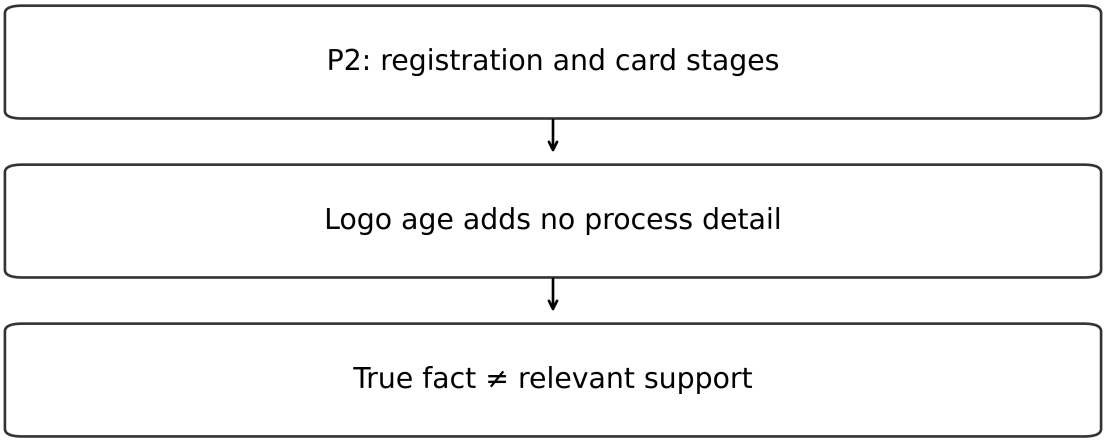
Purpose first

- Before an add, delete or replace question, sum up the paragraph's purpose in a few words.
- Ask: what would the reader lose if this sentence were removed?
- The answer choices often give reasons. The right one names the paragraph's actual purpose.

Evidence and support

- Support must match the claim: a count of complaints supports "noise was a problem"; a description of the room does not.
- The strongest support is specific: numbers, examples, named actions.
- A new detail proposed by the writer should be clearly linked to the paragraph's topic.

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Worked steps: P2: registration and card stages → Logo age adds no process detail → True fact ≠ relevant support

Objections and limits

- A counterargument or limit can strengthen writing: it shows the writer has thought about problems.
- Keep an objection if the passage responds to it, or if it explains a later decision.
- A conclusion should match the evidence: if other things changed at the same time, the cause is uncertain.

Practice passage

Original continuous editing passage – A place to mend

[P1] On the first Saturday of each month, a group of volunteers opens a repair table in the public library. [S1] The project began when librarian Noor noticed a row of broken desk lamps outside a nearby building. [S2] Some needed specialist work, but others had loose switches or damaged cables. [S3] Noor did not want untrained visitors to try unsafe electrical repairs. [S4] She invited a qualified repairer to explain which items the group could inspect and which should go to a professional. The first session accepted only battery-powered objects, with damaged batteries excluded. Visitors registered their items and described the faults before any work began.

[P2] At the entrance, two volunteers record each visitor's name and item. This record helps the group reunite objects with their owners. A blue card goes beside an object awaiting inspection; a white card marks one ready for collection. The cards do not certify safety or guarantee a repair. They simply show where an item is in the process. [S5] The library's logo was designed many years ago. Beside the cards, a sign asks visitors to stay while their object is examined. Staying allows owners to explain an intermittent fault that might not appear immediately. It also gives them a chance to

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learn a simple maintenance habit.

[P3] Space soon became a problem. The repair table shared an alcove with readers who wanted a quiet place to work. During the first two sessions, a visitor log recorded six complaints about conversation near the alcove. Noor proposed moving the table into a meeting room. The room could hold fewer people, so a move alone would not solve the problem. The group introduced short arrival windows and kept two spaces for visitors without bookings. [S6] This totally amazing system fixed every problem forever. In fact, one visitor still waited outside, and the group had to improve the signs leading to the room. The log recorded one noise complaint during the next two sessions; attendance also fell, so the group did not attribute the whole reduction to the move.

[P4] Volunteers were careful about how they described success. An object leaving the table might work again, need another part, or require professional attention. They recorded these outcomes separately. During the trial, forty objects were inspected. Eighteen worked after a permitted minor adjustment, twelve needed a part, and ten were referred elsewhere. Counting all forty as repaired would hide important differences. [S7] The volunteers recorded the outcomes and wrote the results down in their record. Noor used the figures to decide which spare parts might be useful at later sessions. She did not use them to promise that any particular object could be repaired.

[P5] The project now has a practical aim: help visitors make an informed next decision about a broken object. For some, that means a working lamp. For others, it means knowing when further work would be unsafe or too costly. The group plans to keep the arrival windows for another trial and ask readers whether noise remains a problem. It will also compare attendance and waiting times before adding more spaces. A repair table succeeds through careful limits as well as useful repairs.

Worked example: deleting and replacing

Should S5 (“The library’s logo was designed many years ago”) remain in P2?

- P2’s purpose: how visitors register and how the cards show each item’s stage.
- The logo adds nothing to that purpose, so delete it.
- A relevant replacement might explain how a volunteer changes a card when an inspection finishes. Since the passage does not state this, it should be offered as a suggestion, not a fact.

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Worked example: support, objection and conclusion

- Evidence that noise was a problem: "a visitor log recorded six complaints" in the first two sessions.
- "The room could hold fewer people" is an objection to moving. Keep it: it explains why the group also introduced arrival windows and kept walk-in spaces.
- A qualified conclusion: "Recorded complaints fell from six to one in the next two sessions, but attendance also fell, so the move's separate effect is uncertain."
- "The move eliminated noise" fails: one complaint remained, and other conditions changed.

Common errors

- Keeping a sentence because it is true or interesting.
- Choosing support that matches the topic but not the claim.
- Deleting an objection that the passage later answers.
- Writing a conclusion stronger than the evidence.
- Presenting a suggested detail as a fact from the passage.

Key terms

relevance: How directly a sentence serves its paragraph's purpose.

detail: A specific fact or example.

evidence: Information that supports a claim.

counterargument: A point against the writer's position.

argument: A claim with reasons that support it.

Check yourself

- A paragraph argues that a school garden teaches science. Which supports it better: "Students measured plant growth each week" or "The garden is next to the gym"?
- Why might a writer include "Some parents worried about the cost"?
- Complaints fell after a change, but fewer people came. Is "The change ended complaints" justified?

Answers: the first. To answer the objection and strengthen the argument. No: complaints did not reach zero, and the cause is uncertain.

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English: paragraph order, cohesion and introductions

ACT

ACT English: Paragraph Order, Cohesion and Introductions

In September 1787 the Constitutional Convention in Philadelphia had agreed on the content of a new American constitution, spread over 23 articles. It handed them to a five-man Committee of Style. Gouverneur Morris, who did most of the drafting, reorganised the text in a few days into seven articles, and wrote a new opening: "We the People of the United States, in Order to form a more perfect Union…". The ideas hardly changed; the order and the links between them did. ACT English tests this kind of editing: where a sentence belongs, and how ideas connect.

These questions ask you to:

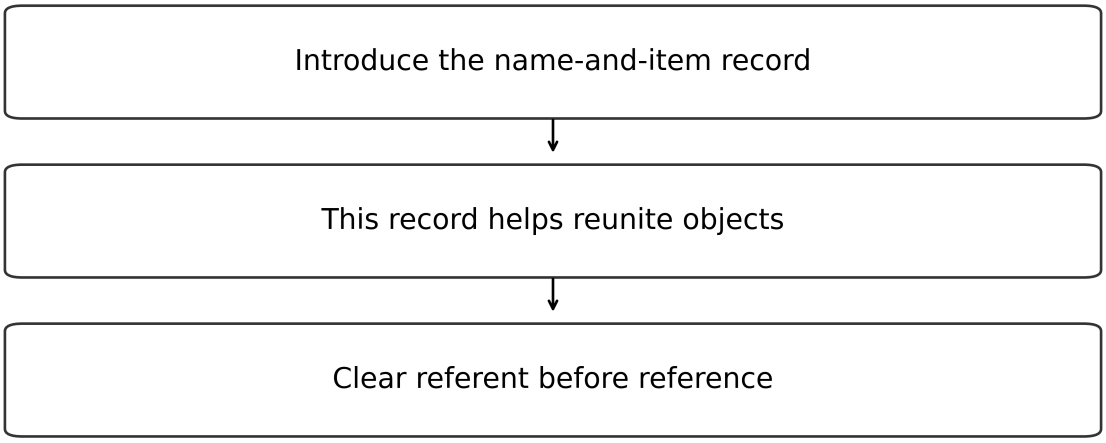
- Decide the best order of sentences and paragraphs.
- Choose the best **location** 位置 for a new sentence.
- Give every reference word a clear **antecedent** 先行词.
- Build **cohesion** 衔接 with a fitting **transition** 衔接词, and write effective introductions and conclusions.

Order and placement

- Introduce a person, place or thing before referring to it.
- Follow time order: words like "soon", "later" and "then" assume something came before.
- Put a sentence beside the one it explains, gives an example of, or answers.

Reference words

- "This", "these", "it", "they" and "such" point back. The thing they point to must be clear and close.
- "This record" needs a record mentioned just before. If two things could be meant, name the right one.



Worked steps: Introduce the name-and-item record → This record helps reunite objects → Clear referent before reference

Transitions between sentences and paragraphs

- Choose the transition by the relationship: addition, contrast, cause, example or time.
- A paragraph's first sentence often links back to the previous paragraph and forward to its own topic.

Introductions and conclusions

- An introduction sets up the topic and prepares for what follows.
- A conclusion returns to the main point and may look ahead, but does not add a new topic or a promise the passage cannot support.

Practice passage

Original continuous editing passage – A place to mend

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[P2] At the entrance, two volunteers record each visitor's name and item. This record helps the group reunite objects with their owners. A blue card goes beside an object

awaiting inspection; a white card marks one ready for collection. The cards do not certify safety or guarantee a repair. They simply show where an item is in the process. [S5] The library's logo was designed many years ago. Beside the cards, a sign asks visitors to stay while their object is examined. Staying allows owners to explain an intermittent fault that might not appear immediately. It also gives them a chance to learn a simple maintenance habit.

[P3] Space soon became a problem. The repair table shared an alcove with readers who wanted a quiet place to work. During the first two sessions, a visitor log recorded six complaints about conversation near the alcove. Noor proposed moving the table into a meeting room. The room could hold fewer people, so a move alone would not solve the problem. The group introduced short arrival windows and kept two spaces for visitors without bookings. [S6] This totally amazing system fixed every problem forever. In fact, one visitor still waited outside, and the group had to improve the signs leading to the room. The log recorded one noise complaint during the next two sessions; attendance also fell, so the group did not attribute the whole reduction to the move.

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Worked example: could P3 come first?

- P3 refers to "the repair table" and to Noor as though the reader already knows them; P1 introduces both.
- P3 begins "Space soon became a problem": "soon" assumes the project has already started.
- So P3 cannot come before P1 without rewriting those introductions. The existing order is clearer.

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Worked example: placing a sentence and linking two ideas

Where should "This record helps the group reunite objects with their owners" go?

- After P2's first sentence, which describes the name-and-item record: "This record" then has a close antecedent.
- Before P1: no record has been mentioned. At the end of P5: the reference is far away and interrupts the conclusion.
- Linking "The meeting room reduced noise around the alcove" and "it held fewer visitors": the second limits the first, so use contrast: "…around the alcove. However, it held fewer visitors."
- A fitting closing sentence: "The next trial should judge access, waiting and safety as well as the number of repairs."

Common errors

- Placing a sentence before the thing it refers to.
- Leaving "this" or "it" with two possible antecedents.
- Choosing a transition without checking the relationship.
- Ending with a new topic or an unsupported guarantee.
- Moving a paragraph without checking its time words.

Key terms

location: The place in a passage where a sentence fits best.

antecedent: The word or phrase a reference word points back to.

cohesion: The way sentences connect so that a text reads as one whole.

transition: A word or phrase showing how two ideas relate.

Check yourself

- A sentence begins "These changes saved time." What must come before it?
- Which word in "Later, the group added signs" shows it cannot open the passage?
- Is "And so the library will never have problems again" a good conclusion?

Answers: a description of the changes. "Later". No: it is an unsupported guarantee.

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English: precision, concision and consistent style

ACT

ACT English: Precision, Concision and Consistent Style

On 9 August 1940, during the most dangerous months of the Second World War, Winston Churchill sent a one-page memo to his government departments. Its title was "Brevity". He asked for reports "in a series of short, crisp paragraphs", and for officials to stop using phrases such as "it is also of importance to bear in mind the following considerations". Clear, short writing, he said, would save time and help thinking. The ACT rewards the same qualities: say exactly what is true, in as few words as the meaning needs, in a steady tone.

These questions ask you to:

- Choose **concision** 简洁 by removing **redundancy** 赘余.
- Keep the passage's **register** 语域, usually a **formal register** 正式语域.
- Choose **diction** 措辞 that is precise and accurate to the facts.
- Recognise when a shorter option loses meaning.

Concision

- Cut words that repeat a meaning: "recorded … and wrote down in their record", "past history", "return back".
- Do not cut information. Concision removes repetition, not facts.
- If two options mean the same, the shorter is usually right.

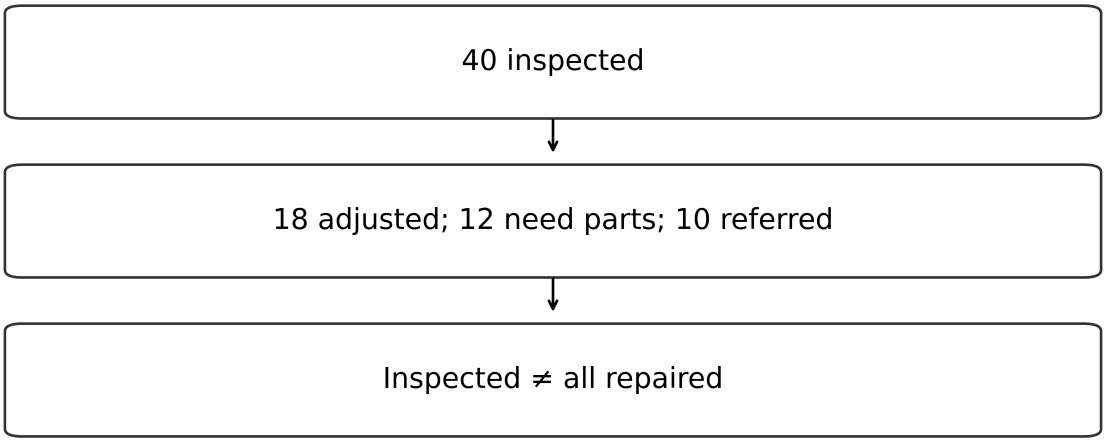
Register and tone

- Most ACT passages are calm and informative. Slang and exaggeration ("totally amazing", "super") break the register.
- Keep the tone consistent from start to finish.
- An exaggeration can also be false: "fixed every problem forever".

Precise diction

- Choose the word that matches the facts. "Inspected" and "repaired" are not the same.
- Prefer exact numbers and actions to vague words like "things" or "stuff".
- A word that sounds more impressive is not better if it says something untrue.

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Worked steps: 40 inspected → 18 adjusted; 12 need parts; 10 referred → Inspected ≠ all repaired

Practice passage

Original continuous editing passage – A place to mend

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Worked example: concise and complete

S7: “The volunteers recorded the outcomes and wrote the results down in their record.”

- “Recorded” and “wrote the results down in their record” say the same thing.
- Concise version: “The volunteers recorded the outcomes.”
- Deleting the whole sentence would go too far: it would remove the fact that they kept a record.

Worked example: register and precision

- S6: “This totally amazing system fixed every problem forever.” “Totally amazing” is informal; “every problem forever” is contradicted by the next sentence.
- Revision: “The arrival windows improved scheduling, but one visitor still waited outside.”
- “Forty objects were inspected” cannot become “forty objects were repaired”: only 18 worked after an adjustment, 12 needed parts and 10 were referred elsewhere. Precision decides, not length.

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Common errors

- Choosing a longer option that repeats a meaning.
- Cutting a sentence that contains needed information.
- Accepting slang or exaggeration in a formal passage.
- Choosing a shorter word that changes the facts.
- Choosing a vague word when a precise one is available.

Key terms

concision: Using no more words than the meaning needs.

redundancy: Repeating a meaning without need.

register: The level of formality of writing.

formal register: A serious, careful style without slang.

diction: The choice of words.

Check yourself

- Make concise: "In my personal opinion, I think the plan is good."
- Which fits a science report: "The results were awesome" or "The results supported the hypothesis"?
- A writer has 30 tested samples, of which 20 passed. Can they write "30 samples passed"?

Answers: "I think the plan is good." The second. No: only 20 passed.

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English: sentence formation, agreement and punctuation

ACT

ACT English: Sentence Formation, Agreement and Punctuation

In 1872 the US Congress passed a tariff law with a list of goods that could enter the country tax-free. It was meant to include “fruit-plants”, young plants for growing fruit. But a clerk copying the bill wrote a comma instead of the hyphen: “fruit, plants”. Importers argued that all foreign fruit was now tax-free, and the Treasury had to agree. The comma is said to have cost the government about 2 million dollars in lost taxes before Congress corrected the law in 1874. The **conventions** 语言规范 of punctuation are rules about meaning, and ACT English tests them in almost every passage.

These questions ask you to:

- Build complete sentences, each with an **independent clause** 独立分句 and a **finite verb** 限定动词.
- Use a full stop, **semicolon** 分号, comma plus conjunction or **colon** 冒号 correctly between clauses.
- Make verbs and pronouns agree with what they refer to.
- Place each **modifier** 修饰语 next to what it describes, and keep lists parallel.

Complete sentences

- A sentence needs a subject and a finite verb (a verb with tense): “The cards show the stage.”
- “-ing” forms and “to” forms alone are not finite: “The volunteers recording names.” is a fragment.
- A dependent clause (“because the room was small”) cannot stand alone.

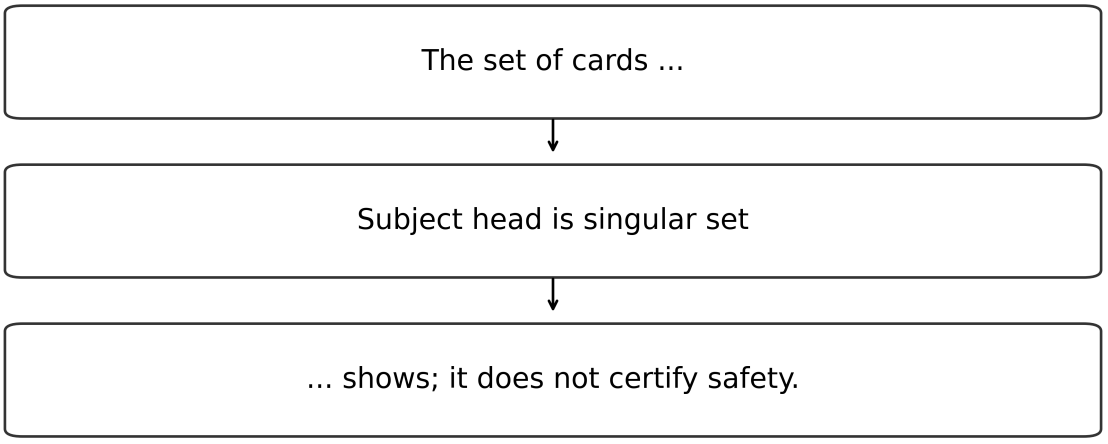
Joining clauses

- Full stop or semicolon between two independent clauses.
- Comma plus and, but, or, so, yet, for or nor.
- Colon after a complete clause, before an explanation, list or example.
- Never a comma alone between two independent clauses.

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Agreement

- Subject–verb: find the true subject. “The set of blue cards shows…”
- Pronoun–noun: “The group updates its records.”
- “Its” is possessive; “it’s” means “it is” or “it has”.



Worked steps: The set of cards … → Subject head is singular set → … shows; it does not certify safety.

Modifiers and parallel lists

- An opening phrase describes the subject right after the comma.
- Keep list items in one form: “to inspect, record and advise”.

Practice passage

Original continuous editing passage – A place to mend

[P1] On the first Saturday of each month, a group of volunteers opens a repair table in the public library. [S1] The project began when librarian Noor noticed a row of broken desk lamps outside a nearby building. [S2] Some needed specialist work, but others had loose switches or damaged cables. [S3] Noor did not want untrained visitors to try unsafe electrical repairs. [S4] She invited a qualified repairer to explain which items the group could inspect and which should go to a professional. The first session accepted only battery-powered objects, with damaged batteries excluded. Visitors registered their items and described the faults before any work began.

[P2] At the entrance, two volunteers record each visitor’s name and item. This record helps the group reunite objects with their owners. A blue card goes beside an object awaiting inspection; a white card marks one ready for collection. The cards do not certify safety or guarantee a repair. They simply show where an item is in the process.

[S5] The library's logo was designed many years ago. Beside the cards, a sign asks visitors to stay while their object is examined. Staying allows owners to explain an intermittent fault that might not appear immediately. It also gives them a chance to learn a simple maintenance habit.

[P3] Space soon became a problem. The repair table shared an alcove with readers who wanted a quiet place to work. During the first two sessions, a visitor log recorded six complaints about conversation near the alcove. Noor proposed moving the table into a meeting room. The room could hold fewer people, so a move alone would not solve the problem. The group introduced short arrival windows and kept two spaces for visitors without bookings. [S6] This totally amazing system fixed every problem forever. In fact, one visitor still waited outside, and the group had to improve the signs leading to the room. The log recorded one noise complaint during the next two sessions; attendance also fell, so the group did not attribute the whole reduction to the move.

[P4] Volunteers were careful about how they described success. An object leaving the table might work again, need another part, or require professional attention. They recorded these outcomes separately. During the trial, forty objects were inspected. Eighteen worked after a permitted minor adjustment, twelve needed a part, and ten were referred elsewhere. Counting all forty as repaired would hide important differences. [S7] The volunteers recorded the outcomes and wrote the results down in their record. Noor used the figures to decide which spare parts might be useful at later sessions. She did not use them to promise that any particular object could be repaired.

[P5] The project now has a practical aim: help visitors make an informed next decision about a broken object. For some, that means a working lamp. For others, it means knowing when further work would be unsafe or too costly. The group plans to keep the arrival windows for another trial and ask readers whether noise remains a problem. It will also compare attendance and waiting times before adding more spaces. A repair table succeeds through careful limits as well as useful repairs.

Worked example: the passage gets it right

The passage itself shows several correct patterns.

- P2: "A blue card goes beside an object awaiting inspection; a white card marks one ready for collection." Two independent clauses, joined by a semicolon.
- P5: "The project now has a practical aim: help visitors make an informed next decision…" A complete clause, then a colon introducing the aim.
- P1: "Some needed specialist work, but others had loose switches or damaged cables." Comma plus "but" between two independent clauses.

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Worked example: three repairs

- "The set of blue cards show the current stage, they do not certify safety." → "The set of blue cards shows the current stage; it does not certify safety." (singular "set"; a comma splice fixed with a semicolon).
- "Inspecting the lamp, a loose switch was noticed by Noor." → "Inspecting the lamp, Noor noticed a loose switch."
- "The group aims to inspect, recording, and advise." → "to inspect, record, and advise". And: "The group updates its records"; "It's important to label outcomes."

Common errors

- Writing a fragment with an "-ing" verb and no finite verb.
- Joining two independent clauses with a comma alone.
- Putting a colon after an incomplete clause.
- Making the verb agree with the nearest noun.
- Confusing "its" and "it's".

Key terms

conventions: The accepted rules of grammar and punctuation.

independent clause: A clause with a subject and finite verb that can stand alone.

finite verb: A verb with tense that can be a clause's main verb.

semicolon: A mark (;) that joins two closely related independent clauses.

colon: A mark (:) that follows a complete clause and introduces what explains it.

modifier: A word or phrase that describes another word.

Check yourself

- Is "Volunteers checking each item at the door." a complete sentence?
- Correct: "The lamp worked, the fan did not."
- Choose: "Each of the volunteers ____ a card." (has / have)

Answers: no: "checking" is not a finite verb; write "Volunteers check each item at the door." "The lamp worked; the fan did not" or "The lamp worked, but the fan did not." "has".

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Reading: explicit detail, inference and relationships

ACT

ACT Reading: Explicit Detail, Inference and Relationships

In 1822 the French scholar Jean-François Champollion announced that he could read Egyptian hieroglyphs. His key was the Rosetta Stone, found in 1799, which repeats one decree in Greek and in two Egyptian scripts. Champollion started from explicit details: certain signs were enclosed in ovals, and the Greek text showed where the royal names Ptolemy and Cleopatra appeared. From the signs those names shared, he inferred their sounds, then tested each guess on other names. Careful inference begins with exactly what is written. ACT Reading tests both steps.

These questions ask you to:

- Locate an explicit **detail** 细节 in the passage.
- Draw an **inference** 推断 that the **evidence** 证据 supports.
- Identify relationships: **cause and effect** 因果关系, comparison and **sequence** 顺序.

Explicit details

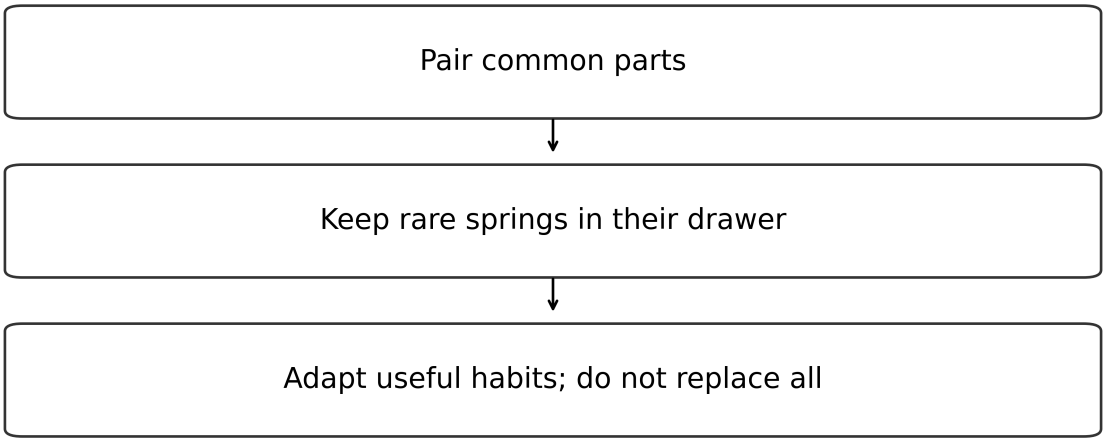
- Find the line first. Many questions give a paragraph number; others need a quick scan for a key word.
- Read a sentence before and after: the answer is often a little away from the key word.
- The right answer usually restates the passage in different words.

Inferences

- An inference joins two or more stated facts.
- Test each option: can you point to the lines that support it?
- Reject options that add numbers, causes, feelings or events the passage does not give.
- In a first-person story, the narrator may say they are unsure. Do not turn their guess into a fact.

Relationships

- Cause and effect: look for “because”, “so”, “after”, “which made”.
- Comparison: what is the same and what differs between two people, places or methods?
- Sequence: put events in the order they happened, which may differ from the order they are told.



Worked steps: Pair common parts → Keep rare springs in their drawer → Adapt useful habits; do not replace all

Practice passage

Original literary passage – The spare key

When I returned to the workshop after my uncle retired, the key felt too small for the responsibility attached to it. He had left the drawers neatly labelled, as though the labels might continue his explanations in his absence. I turned on the overhead light and read “washers”, “pins”, “springs”. Each word named an orderly compartment. The workbench, however, carried a shallow circular mark where his tea cup had stood, and no label could explain why that empty circle held my attention longer than the stocked shelves.

A neighbour arrived with a bent gate latch. I found a replacement pin quickly, then spent several minutes searching for the washer that would keep it in place. My uncle had sorted objects by size. He could see a repair in his mind before he opened a drawer; I could not. The neighbour waited without complaint, which made the delay feel more noticeable rather than less. At last I found the washer, fitted the pin and watched the latch swing freely. The repair was simple. Finding its parts had not been.

That evening I placed commonly paired pieces in small trays. The shelves lost some of their symmetry. I imagined my uncle raising an eyebrow, then remembered how often

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he had moved a tool after noticing where his hand expected it to be. His neat labels had been the result of change, not a prohibition against it. I left the rare springs in their old drawer because they had no regular partners. There was no reason to change them merely to prove that the workshop was mine.

A week later my uncle visited. He opened a tray and asked where the larger washers had gone. I showed him the remaining size-labelled drawer. "So you have kept both systems," he said. I could not tell whether his smile meant approval or amusement at my earnestness. He tested a latch, set it down, and asked for tea. I placed his cup on the old mark without thinking. Only after he left did I notice that I had made room beside it for my own.

I had expected taking charge to mean replacing his habits with mine. Instead, I was learning which habits solved a problem and which had depended on a skill I had not yet acquired. The workshop did not need a dramatic beginning. It needed a washer that could be found, a safe repair, and enough empty space for the next person to work.

Worked example: a cause and a sequence

Why does the narrator start pairing parts?

- Cause (paragraph 2): a simple latch repair took a long time because the narrator had to search for the matching washer separately.
- Explicit contrast: the uncle "could see a repair in his mind before he opened a drawer; I could not."
- Sequence: the repair happens; "that evening" the narrator pairs common parts; "a week later" the uncle visits.
- The passage states no measured time saved, so an answer claiming "repairs became twice as fast" adds a number.

Worked example: a comparison and its limit

What stays in the old arrangement, and why?

- The rare springs stay in their old drawer "because they had no regular partners"; a size-labelled drawer for larger washers also remains.
- So the narrator keeps parts of both systems. The uncle says it: "So you have kept both systems."
- Limit: "I could not tell whether his smile meant approval or amusement." Any answer stating that the uncle approved goes beyond the evidence.

Common errors

- Answering from memory instead of the line.
- Choosing an inference that adds a number or a feeling.
- Confusing the order of telling with the order of events.

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- Treating a narrator's uncertainty as a fact.
- Choosing an answer that is true for one paragraph but contradicted by another.

Key terms

detail: A specific piece of information stated in the passage.

inference: A conclusion supported by stated information.

evidence: The lines that support an answer.

cause and effect: A relationship in which one event leads to another.

sequence: The order in which events happen.

Check yourself

- What does the narrator do immediately after the neighbour's repair?
- Why did the narrator search so long for the washer?
- Does the passage say the uncle disliked the changes?

Answers: that evening, they place commonly paired pieces in small trays. The uncle had sorted objects by size, and the narrator could not picture the repair in advance. No: the narrator cannot tell what his smile meant.

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Reading: contextual meaning, structure and point of view

ACT

ACT Reading: Contextual Meaning, Structure and Point of View

"Reader, I married him." Charlotte Brontë's novel *Jane Eyre*, published in London in October 1847 under the name "Currer Bell", is told by Jane herself, looking back on her life. Because the narrator speaks directly, readers know her thoughts and see others only through her eyes, and that changes everything they understand about the story. Who is telling a story, what their words suggest, and how the story is built are the questions of this part of ACT Reading.

These questions ask you to:

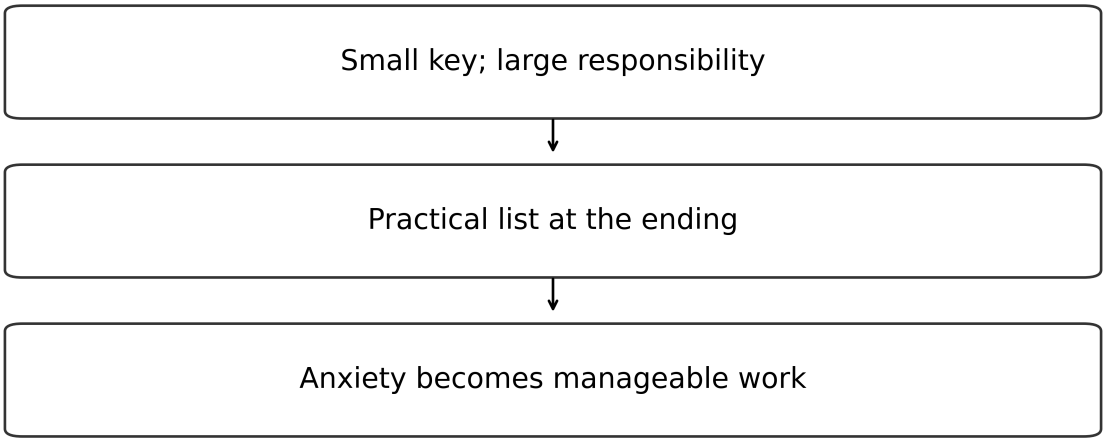
- Explain the meaning of a word or phrase in context, including its **connotation** 内涵色彩 and any **figurative language** 比喻性语言.
- Describe how a passage is structured and what a part of it does.
- Identify the **point of view** 叙述视角 and the **tone** 语气.
- Support each answer with **evidence** 证据 from the text.

Meaning in context

- Read the whole sentence, and the one before it, before choosing a meaning.
- Figurative language compares or imagines: "as though the labels might continue his explanations" does not mean the labels talk.
- Connotation is the feeling a word adds: "earnestness" suggests seriousness, perhaps a little too much.

Structure

- Track how the passage changes: a feeling at the start, an event, a response, a reflection.
- Endings often answer or change the opening. Compare the first and last paragraphs.
- Ask what a scene does for the story: shows a problem, a relationship, a change.



Worked steps: Small key; large responsibility → Practical list at the ending → Anxiety becomes manageable work

Point of view and tone

- A first-person narrator knows their own thoughts but can only guess at others'.
- A narrator looking back may understand more than they did at the time.
- Tone comes from word choice and can be mixed: warm and uncertain, amused and respectful. Avoid extreme answers unless the words are extreme.

Practice passage

Original literary passage – The spare key

When I returned to the workshop after my uncle retired, the key felt too small for the responsibility attached to it. He had left the drawers neatly labelled, as though the labels might continue his explanations in his absence. I turned on the overhead light and read "washers", "pins", "springs". Each word named an orderly compartment. The workbench, however, carried a shallow circular mark where his tea cup had stood, and no label could explain why that empty circle held my attention longer than the stocked shelves.

A neighbour arrived with a bent gate latch. I found a replacement pin quickly, then spent several minutes searching for the washer that would keep it in place. My uncle had sorted objects by size. He could see a repair in his mind before he opened a drawer; I could not. The neighbour waited without complaint, which made the delay feel more noticeable rather than less. At last I found the washer, fitted the pin and watched the latch swing freely. The repair was simple. Finding its parts had not been.

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A week later my uncle visited. He opened a tray and asked where the larger washers had gone. I showed him the remaining size-labelled drawer. "So you have kept both systems," he said. I could not tell whether his smile meant approval or amusement at my earnestness. He tested a latch, set it down, and asked for tea. I placed his cup on the old mark without thinking. Only after he left did I notice that I had made room beside it for my own.

I had expected taking charge to mean replacing his habits with mine. Instead, I was learning which habits solved a problem and which had depended on a skill I had not yet acquired. The workshop did not need a dramatic beginning. It needed a washer that could be found, a safe repair, and enough empty space for the next person to work.

Worked example: figurative wording

What does "as though the labels might continue his explanations in his absence" suggest?

- "As though" marks an imagined comparison, not a fact.
- The labels seem to carry on the uncle's teaching after he has retired.
- This prepares a contrast: the labelled drawers explain objects, but nothing labels the cup mark, which holds the narrator's attention for a personal reason.
- In paragraph 1, "the key felt too small for the responsibility" contrasts a small object with a large feeling.

Worked example: structure and tone

- Opening: a small key and a large, vague responsibility. Ending: a practical list, "a washer that could be found, a safe repair, and enough empty space for the next person to work".
- The structure moves from anxiety to manageable work, and the narrator's expectation of "replacing his habits" gives way to choosing which habits to keep.
- Tone towards the uncle is not hostile: the narrator remembers how he adapted his tools and makes room for his cup. It is respectful and a little uncertain, since the narrator "could not tell" what his smile meant.

Common errors

- Reading figurative language literally.
- Choosing a word's dictionary meaning without the context.
- Describing a passage's topic when asked for its structure.

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- Giving the narrator knowledge of other characters' thoughts.
- Choosing an extreme tone, such as bitter or joyful, for mixed feelings.

Key terms

connotation: The feeling or idea a word suggests beyond its basic meaning.

figurative language: Words used to compare or imagine, not to state literally.

point of view: Who tells the story and what they can know.

tone: The attitude that word choice shows.

evidence: Words from the text that support an answer.

Check yourself

- In "The shelves lost some of their symmetry", is the narrator mainly praising or describing a cost?
- From whose point of view is the passage told, and what limit does that create?
- What does the final paragraph do in the structure of the passage?

Answers: describing a small cost of the change. The narrator's, in the first person; they can only guess what the uncle thinks. It reflects on the change and reframes "taking charge" as practical work.

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Reading: arguments, paired texts and graphic evidence

ACT

ACT Reading: Arguments, Paired Texts and Graphic Evidence

In 1662 John Graunt, a London cloth merchant, published *Natural and Political Observations upon the Bills of Mortality*. For decades London parishes had printed weekly lists of deaths and their causes, and almost nobody had analysed them. Graunt added them up into tables. He showed, for example, that more boys than girls were born each year, and that plague years could be read clearly in the numbers. He made arguments from data, and he was careful about what his tables could not show. ACT Reading now asks you to combine arguments with tables and graphs in the same way.

These questions ask you to:

- Identify each text's **claim** 主张 and its supporting **argument** 论证.
- Compare two texts: where they agree, and where they differ.
- Show **integration** 综合 of text and data, judging the **relevance** 相关性 of a table to a claim.
- Decide whether the **evidence** 证据 supports, weakens or does not address a claim.

Reading an argument

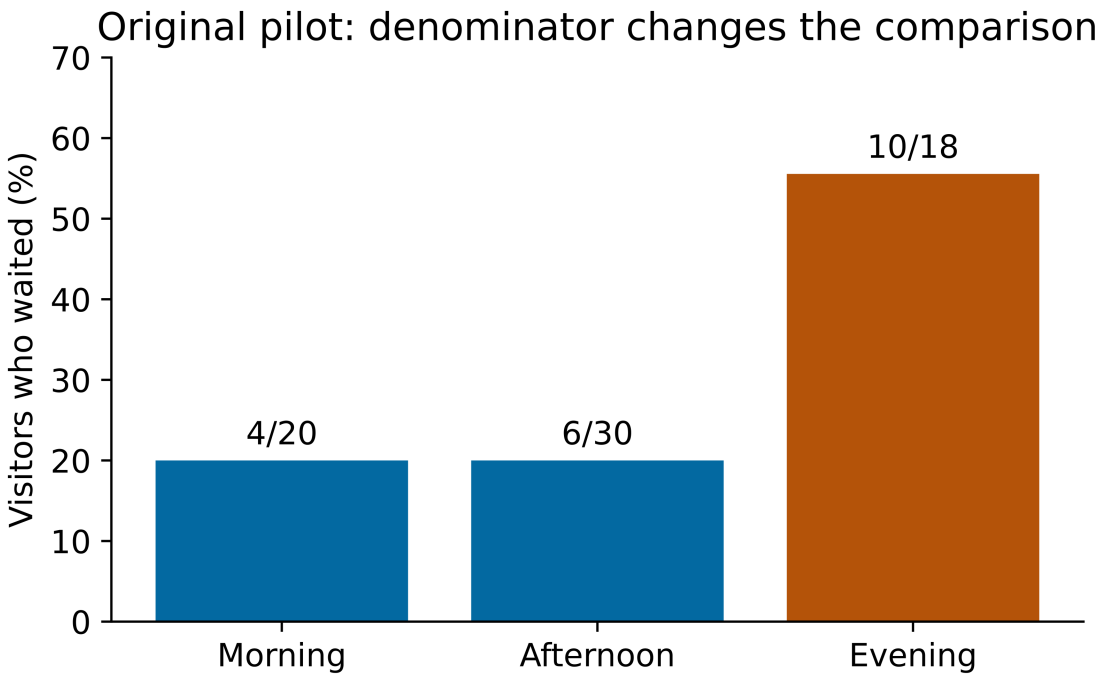
- Find the claim (what the writer wants you to accept), the reasons, and any limits the writer admits.
- Notice what the writer permits as well as what they reject.
- Note signal words: "should", "must", "also", "however".

Comparing two texts

- Summarise each text in one sentence before comparing.
- Texts may share a goal but differ in priorities.
- Ask how each author would respond to a third claim: often both would reject an extreme version.

Reading a graphic

- Read the title, labels, units and notes. Notes often say what the data do not cover.
- Compare rates when groups differ in size, not counts.
- Check the exact claim: "every" or "all" needs every row to agree.



Morning and afternoon both show 20% waiting despite different counts: compare rates, not counts

In the figure: $4/20 = 20\%$, $6/30 = 20\%$ and $10/18 \approx 56\%$, so only the evening stands out.

Practice texts and table

Original argumentative Text A – Preserve the labels

A workshop catalogue should preserve the names and categories used by its earlier keepers. These records show how people understood their tools. If each new keeper replaces every label with one chosen only for current convenience, later readers lose evidence of that earlier understanding. Preservation does not require freezing the room. A new index can connect old categories to present uses, while a record of each change lets readers reconstruct the earlier arrangement. Convenience matters, but a useful system need not erase its history.

Original argumentative Text B – Let the search task guide the index

An index is also a tool used by people who may lack the keeper's experience. In a trial with beginners, pairing commonly used parts reduced the average time needed to gather a repair kit. The improvement did not extend to the rare parts, which beginners used in several different combinations. We therefore recommend task-based trays for frequent combinations and an ordinary size index for the remainder. Old labels can be recorded separately. This recommendation concerns retrieval under the trial's tasks, not the historical value of the objects or a guarantee about every future repair.

Original hypothetical retrieval table – average minutes per kit, same beginners com-

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pleting matched tasks in counterbalanced order:

- Frequent combinations: size index 8; task-based trays 5.
- Rare combinations: size index 10; task-based trays 11.
- Each average summarises twelve trials; no uncertainty estimates are supplied.

The table does not compare historians, expert keepers, repair quality or object safety. It records retrieval times only.

Worked example: two texts, one arrangement

- Text A's main concern: keeping evidence of how earlier keepers named and grouped their tools. It allows a new index if old labels and changes are recorded.
- Text B's main concern: helping beginners gather parts quickly. It keeps a size index for rare parts.
- A system that satisfies both: keep a dated record of old labels and changes, use task-based trays for frequent combinations, and keep the size index for rare ones.
- Both would reject "all old labels should be discarded" (A for history, B because the size index still helps rare searches); B would also reject "no category should ever change".

Worked example: does the table support the claim?

Claim: "Task-based trays speed every search."

- Frequent kits: $8 \to 5$ minutes, 3 minutes faster ($\frac{3}{8} = 37.5\%$).
- Rare kits: $10 \to 11$ minutes, 1 minute slower (10%).
- The table contradicts "every search". It also gives averages of twelve trials with no uncertainty, and says nothing about repair quality or safety, so it cannot support claims about those.

Common errors

- Merging two authors' views.
- Treating a writer's permitted exception as a rejection.
- Comparing counts for groups of different sizes.
- Ignoring the notes beneath a table.
- Accepting "every" when one row disagrees.

Key terms

claim: A statement the writer wants the reader to accept.

argument: A claim with the reasons that support it.

integration: Combining information from texts and graphics.

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relevance: Whether a piece of information bears on the claim.

evidence: Information that supports or weakens a claim.

Check yourself

- Which text would agree that an index can change if old labels are recorded?
- Does the table show that trays improve repair quality?
- Group X: 6 of 12 waited. Group Y: 9 of 30 waited. Which had the higher rate?

Answers: Text A (and Text B). No: it records retrieval times only. X (50% against 30%).

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Science: research summaries, controls and engineering tests

ACT

ACT Science: Research Summaries, Controls and Engineering Tests

In the autumn of 1901 Wilbur and Orville Wright, bicycle makers in Dayton, Ohio, were disappointed with their gliders: the published tables of wing lift seemed to be wrong. They built a small wind tunnel, about 1.8 metres long, with a fan driven by the shop's engine. Inside, they tested around 200 small model wings, changing one feature at a time (curve, shape, aspect) and measuring lift with balances made from bicycle spokes and hacksaw blades. Their new data led to the 1903 Flyer. ACT research summaries describe experiments and design tests like theirs; your job is to read what was changed, what was measured, and what the results do and do not show.

These questions ask you to:

- Identify the **independent variable** 自变量, the **dependent variable** 因变量 and the controls.
- Judge whether a **controlled comparison** 控制比较 is fair.
- Evaluate an **engineering design** 工程设计 against each **constraint** 约束条件.
- Propose or evaluate a **proposal** 提案 for a follow-up test.

Variables and controls

- The independent variable is what the researchers change on purpose. The dependent variable is what they measure.
- Controlled variables are kept the same so that only one factor differs.
- A control (such as an unsleeved sensor) shows what happens with no treatment.

Repeats and ranges

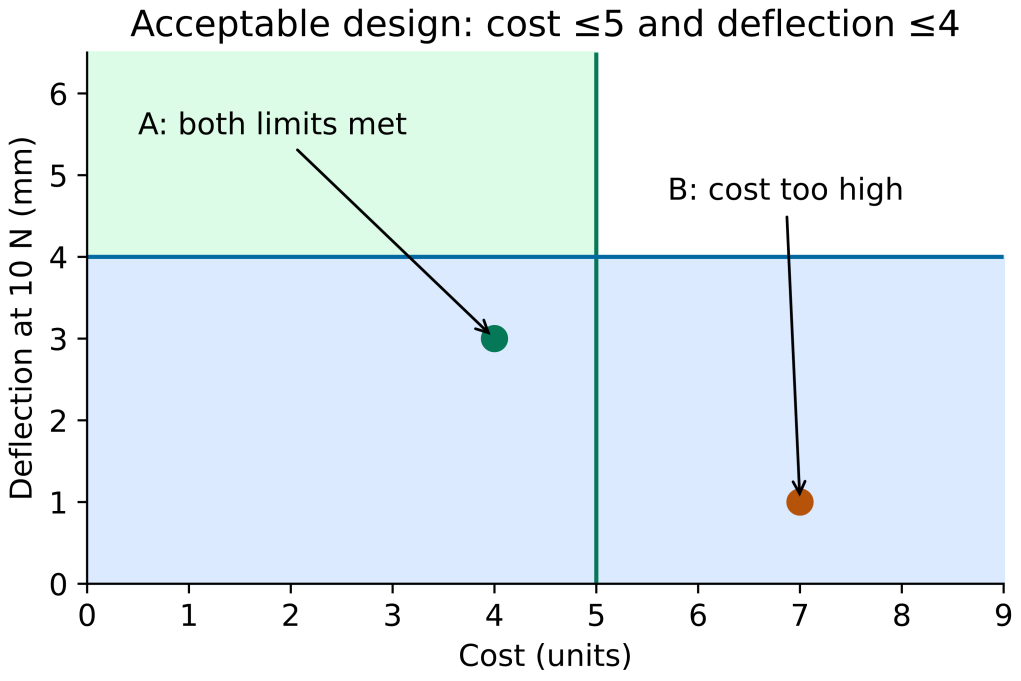
- Repeated trials show how much results vary. Report the mean and the range.
- A range describes these trials, not every future result.
- If two ranges overlap a lot, a small difference in means may not be meaningful.

Engineering tests

- A design brief lists constraints, such as maximum cost and maximum temperature.
- A design is acceptable only if it meets every constraint. The best performance on one measure is not enough.

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- Requirements not yet measured, such as durability, still need testing.



A design is acceptable only inside both limits: cost at most 5 and deflection at most 4

Planning a follow-up

- Change only the factor you want to study; hold the others.
- Measure any requirement that a change might affect, for example the cost of a thicker sleeve.
- State acceptance limits before testing, not after.

Practice research summary

Original hypothetical research summary – A cooling sleeve

A laboratory team tests sleeves intended to reduce a small sensor's temperature during operation. All sensors start at 22°C. Temperature is measured after ten minutes of use with the same power setting, room conditions and measuring instrument. Sleeve A costs 4 units, B costs 7 units, and an unsleeved sensor is the control. The design brief requires a cost no greater than 5 units and a final temperature no greater than 35°C. Lower temperature alone is not the only design criterion.

Experiment 1 uses identical sensors and records three independent trials per sleeve. Final temperatures in °C are: control 41,42,43; A 33,34,35; B 30,31,32. The team resets the starting temperature and checks the sensor charge before each trial. They report the means and the full observed ranges. The ranges describe these trials, not

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every future result. The team has not yet measured sleeve durability.

Experiment 2 investigates thickness for material A. Each sleeve has the same length and fit; thicknesses are 1,2,3 mm. Mean final temperatures are 38,34,32°C respectively. All other stated conditions match Experiment 1. Changing thickness also changes material quantity, so a future design review must measure costs instead of assuming they stay at 4 units. A student proposes testing a 3-mm sleeve in material B against a 1-mm sleeve in material A to identify thickness's effect. This comparison would change two factors at once.

Two original models explain the observed temperature pattern. Model P says final temperature depends only on thickness, regardless of material. Model Q says both thickness and material matter; at equal thickness it predicts that material B produces a lower final temperature than A under the same conditions. Both models predict lower final temperatures as thickness rises over the tested 1–3 mm interval. A decreasing trend alone therefore cannot separate them.

Experiment 3 tests new A and B sleeves, each 2 mm thick, in matched conditions. Mean final temperatures are A 34°C and B 31°C. The measuring instrument's stated resolution is 0.1°C, and the team checks its calibration with a reference. Repeated trials would still be needed to characterise variability and rule out other differences in manufacture. The result conflicts with P's material-independence prediction and is consistent with Q's direction; it is not proof that Q is the only possible explanation.

A separate time series for one sensor with sleeve A records temperatures 22,28,32,34°C at 0,2,4,6 minutes. These observations show warming with successively smaller two-minute increases. No measurements beyond six minutes are supplied in this series. A proposal to continue the first interval's slope to twelve minutes is a model assumption that later measurements may contradict, not a reading from the table.

Worked example: checking the brief

Using Experiment 1:

- Control: mean 42 °C (41–43). A: mean 34 °C (33–35). B: mean 31 °C (30–32).
- The ranges of A and B do not overlap, so B was cooler in every trial.
- A meets both constraints: cost $4 \leq 5$ and temperature ≤ 35 °C in all trials. B fails on cost: $7 > 5$.
- Durability has not been measured, so A is the only candidate so far, not a final choice.

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Worked example: a fair follow-up

- Experiment 2: thickness is the independent variable; final temperature is the dependent variable; material, length, fit, start temperature and conditions are controlled.
- The student's proposal (3 mm of B against 1 mm of A) changes two factors at once, so it cannot isolate thickness.
- A better follow-up for the 3 mm A sleeve: measure its actual cost and durability, and repeat matched temperature tests with the same material, length, fit, power, start temperature and duration. Thicker sleeves use more material, so the cost cannot be assumed to stay at 4 units.

Common errors

- Swapping the independent and dependent variables.
- Choosing the best-performing design while it breaks a constraint.
- Accepting a comparison that changes two factors.
- Treating a range from three trials as a guarantee.
- Ignoring a requirement that has not been measured.

Key terms

independent variable: The factor changed on purpose.

dependent variable: The quantity measured.

controlled comparison: A comparison in which only one factor differs.

engineering design: Developing a solution that meets stated requirements.

constraint: A limit that a design must satisfy.

proposal: A suggested test or design.

Check yourself

- In Experiment 1, what is the purpose of the unsleeved sensor?
- Why can't a 3 mm A sleeve be assumed to cost 4 units?
- A design is cheap and cool but breaks after one day. Does it meet a brief that includes durability?

Answers: it is the control, for comparison. It uses more material, so its cost must be measured. No: it fails one constraint.

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Science: conflicting viewpoints and evidence that separates models

ACT

ACT Science: Conflicting Viewpoints and Evidence That Separates Models

In January 1912 the German meteorologist Alfred Wegener proposed that the continents had once been joined and had slowly drifted apart. He pointed to the matching coastlines of Africa and South America, and to the same fossils on both sides of the Atlantic. Most geologists rejected the idea, partly because he could not explain what moved the continents; a land bridge that later sank seemed to explain the fossils just as well. The question was settled only in the 1960s, when magnetic stripes in the rock of the ocean floor were found to be arranged symmetrically on either side of mid-ocean ridges, as spreading predicted and the older view did not. Evidence that separates two models is evidence where they predict different things.

These questions ask you to:

- State what each model or scientist predicts.
- Recognise each **model assumption** 模型假设.
- Identify **discriminating evidence** 区分性证据: an observation that fits one model but not the other.
- Design or recognise a **discriminating test** 鉴别性检验, and separate a design decision from a test of a **prediction** 预测.

Summarise each viewpoint

- For each model, write one line: what it assumes, and what it predicts.
- Note where they agree. Shared predictions cannot decide between them.
- Note where they differ. That is where a test can decide.

Evidence that separates models

- If both models predict a result, observing it supports neither over the other.
- If they predict different results, the observation can weaken one: “conflicts with P, consistent with Q”.
- One result rarely proves a model; other explanations may still fit.

Different predictions enable a discriminating test

Model	Same runtime; compare 15°C and 30°C
A: time only	Equal brightness
B: temperature rise	Lower brightness at 30°C

A test can decide between two models only where they predict different results

Designing a separating test

- Choose conditions where the predictions differ.
- Change only that factor; hold everything else, and repeat the trials.
- Check that the instrument is precise and calibrated enough to see the difference.

Explanations and decisions

- Whether a design meets a brief is a practical question. Why it works is a scientific one.
- A design can pass every requirement without telling you which model is right.

Practice research summary

Original hypothetical research summary – A cooling sleeve

A laboratory team tests sleeves intended to reduce a small sensor's temperature during operation. All sensors start at 22°C. Temperature is measured after ten minutes of use with the same power setting, room conditions and measuring instrument. Sleeve A costs 4 units, B costs 7 units, and an unsleeved sensor is the control. The design brief requires a cost no greater than 5 units and a final temperature no greater than 35°C. Lower temperature alone is not the only design criterion.

Experiment 1 uses identical sensors and records three independent trials per sleeve.

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Final temperatures in °C are: control 41,42,43; A 33,34,35; B 30,31,32. The team resets the starting temperature and checks the sensor charge before each trial. They report the means and the full observed ranges. The ranges describe these trials, not every future result. The team has not yet measured sleeve durability.

Experiment 2 investigates thickness for material A. Each sleeve has the same length and fit; thicknesses are 1,2,3 mm. Mean final temperatures are 38,34,32°C respectively. All other stated conditions match Experiment 1. Changing thickness also changes material quantity, so a future design review must measure costs instead of assuming they stay at 4 units. A student proposes testing a 3-mm sleeve in material B against a 1-mm sleeve in material A to identify thickness's effect. This comparison would change two factors at once.

Two original models explain the observed temperature pattern. Model P says final temperature depends only on thickness, regardless of material. Model Q says both thickness and material matter; at equal thickness it predicts that material B produces a lower final temperature than A under the same conditions. Both models predict lower final temperatures as thickness rises over the tested 1–3 mm interval. A decreasing trend alone therefore cannot separate them.

Experiment 3 tests new A and B sleeves, each 2 mm thick, in matched conditions. Mean final temperatures are A 34°C and B 31°C. The measuring instrument's stated resolution is 0.1°C, and the team checks its calibration with a reference. Repeated trials would still be needed to characterise variability and rule out other differences in manufacture. The result conflicts with P's material-independence prediction and is consistent with Q's direction; it is not proof that Q is the only possible explanation.

A separate time series for one sensor with sleeve A records temperatures 22,28,32,34°C at 0,2,4,6 minutes. These observations show warming with successively smaller two-minute increases. No measurements beyond six minutes are supplied in this series. A proposal to continue the first interval's slope to twelve minutes is a model assumption that later measurements may contradict, not a reading from the table.

Worked example: predictions and Experiment 3

- Model P: temperature depends only on thickness. So equal-thickness A and B sleeves should give the same temperature.
- Model Q: material matters too; at equal thickness, B should be cooler than A.
- Experiment 2's falling trend fits both models, so it cannot separate them.
- Experiment 3 (both 2 mm): A 34 °C, B 31 °C. A 3 °C difference with a 0.1 °C instrument, calibrated, conflicts with P and is consistent with Q. More repeats are needed to measure variation and rule out manufacturing differences.

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Worked example: a design decision is not a model test

A redesigned B sleeve now costs 5 units and ends at 33 °C in repeated matched tests.

- It meets the brief: cost ≤ 5 and temperature ≤ 35 °C.
- But with no matched A sleeve in the same test, it says nothing new about P against Q.
- So it supports a design decision, not a choice between models. Durability still needs checking.

Common errors

- Treating a result both models predict as support for one.
- Saying one experiment "proves" a model.
- Designing a test that changes two factors.
- Ignoring the instrument's resolution.
- Confusing "the design works" with "the model is right".

Key terms

model assumption: A condition a model takes as true.

prediction: What a model says will happen in a given situation.

discriminating evidence: An observation that fits one model but conflicts with another.

discriminating test: An experiment designed where two models predict different results.

Check yourself

- Both models predict that thicker sleeves are cooler. Can a thickness experiment separate them?
- What comparison would separate P from Q?
- An observation conflicts with P and fits Q. Is Q proved?

Answers: no. Two materials at the same thickness, in matched conditions. No: it is consistent with Q, and other explanations may remain.

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Science: data representation and defensible extrapolation

ACT

ACT Science: Data Representation and Defensible Extrapolation

In 1965 Gordon Moore, an engineer at Fairchild Semiconductor, plotted the number of components on the best computer chips for just a few years and noticed that it was doubling about every year. In an article in *Electronics* magazine he extended the line ten years ahead. In 1975 he revised the rate to doubling about every two years, and the trend, called "Moore's law", held for decades. By the 2010s it was slowing. Moore's prediction was useful because he stated his assumption: that the trend would continue. ACT Science asks you to read data carefully and to say clearly when a value is measured, estimated between points, or extended beyond them.

These questions ask you to:

- Read tables and graphs, with units, and describe a **trend** 趋势.
- Calculate a **rate of change** 变化率 between measurements.
- Estimate between measured points by **interpolation** 插值.
- Recognise **extrapolation** 外推 beyond the data and judge whether the **evidence** 证据 supports it.

Reading representations

- Read the title, axis labels, units and key before the numbers.
- A table gives exact measured values; a graph shows the pattern.
- Describe two things separately: the value (rising, falling) and the rate (speeding up, slowing down).

Rates of change

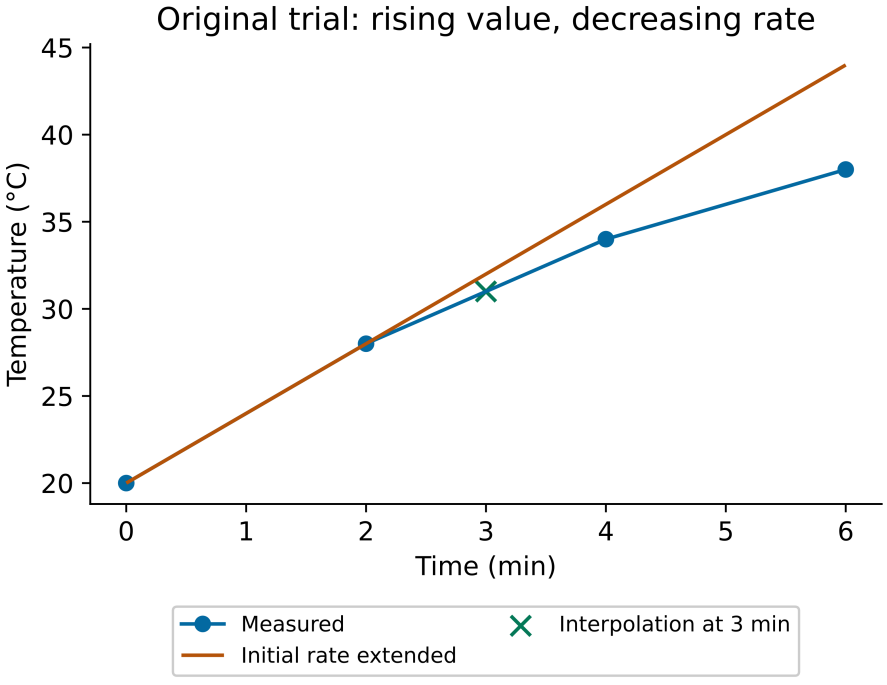
$$\text{average rate} = \frac{\text{change in quantity}}{\text{change in time}}$$

- A rising temperature can have a falling rate of increase: the curve flattens.
- Average rates over intervals do not give the exact rate at every instant.

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Interpolation and extrapolation

- Interpolation estimates between measured points, usually with a straight line between them.
- Extrapolation extends beyond the measured range. It depends on an assumption that the pattern continues.
- Extending the starting rate of a flattening curve overestimates later values.



The temperature keeps rising but more slowly, so extending the starting rate overestimates

In the figure's example the two-minute rises are 8, 6 and 4 °C, so the starting rate of 4 °C per minute would predict 44 °C at 6 minutes, not the measured 38 °C.

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Worked example: rates from the time series

Sleeve A: 22, 28, 32, 34 °C at 0, 2, 4, 6 minutes.

- $\frac{28-22}{2} = 3$, $\frac{32-28}{2} = 2$, $\frac{34-32}{2} = 1$ °C per minute.
- The temperature rises throughout, while the rate of rise falls.
- These are average rates for each interval, not exact rates at each moment.

Worked example: estimating and extending

- Interpolation at 3 minutes: halfway between 28 and 32, so about $28 + \frac{1}{2}(4) = 30$ °C.
- Extending the first rate to 6 minutes: $22 + 3 \times 6 = 40$ °C, but the measurement was 34 °C. The first-slope model overestimates.
- A value at 12 minutes would be an extrapolation beyond the 0–6 minute data. It needs a stated model and more measurements; the rising curve could level off.

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Common errors

- Describing the temperature when the question asks about the rate.
- Reading an extrapolated value as if it were measured.
- Extending a starting rate along a flattening curve.
- Ignoring units on the axes.
- Treating interval averages as exact instantaneous rates.

Key terms

trend: The general direction of change in data.

rate of change: How fast a quantity changes, per unit of time or another variable.

interpolation: Estimating a value between measured points.

extrapolation: Estimating a value beyond the measured range.

evidence: Data that support or weaken a claim.

Check yourself

- Values are 10, 16, 20 at 0, 3, 6 seconds. Find each average rate.
- Is estimating the value at 4.5 seconds interpolation or extrapolation?
- Why is a prediction at 20 seconds less certain?

Answers: 2 and $\frac{4}{3}$ units per second. Interpolation. It is an extrapolation, and the rate is already falling.

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