

Plane transformations, coordinate reasoning and proof

ACT · ACT · ACT

Scope and prerequisites

ACT framework, February 2026 revision. Original classroom cases are not an official form; preserve the scored/field-test boundaries of each source form.

- Transform coordinates while distinguishing rigid motions from dilation
- Use distance, midpoint and slope as geometric evidence
- Use valid congruence and angle relationships rather than a diagram's appearance

Prerequisites: Coordinate differences; triangle congruence; angles on parallel lines.

Explain and choose the method

A translation adds a displacement to each point; reflection across the x-axis sends (x,y) to $(x,-y)$. A 90° counterclockwise rotation around the origin sends (x,y) to $(-y,x)$. These rigid motions preserve lengths and angles. Dilation by factor k about the origin scales coordinates and lengths by k when k is positive, preserving shape but generally changing size.

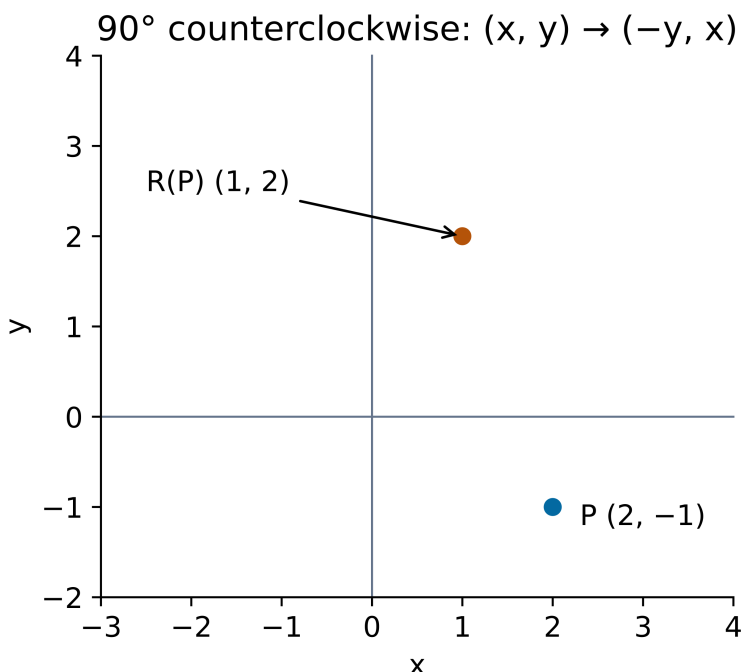
Distance between two points follows the Pythagorean theorem: square the coordinate differences, add and take the root. The midpoint averages each coordinate. Slope compares vertical and horizontal change; perpendicular nonvertical slopes have product -1 . Vertical and horizontal lines require separate treatment rather than division by zero.

A geometric argument starts from stated or marked conditions. Vertical angles are equal; parallel lines give corresponding and alternate-angle relationships. Triangle angles sum to 180° . SSS, SAS and ASA/AAS can establish congruence with proper correspondence, while two equal angles establish similarity but not equal size.

Construction reasoning uses a compass to transfer lengths and draw equal-radius arcs. An intersection of equal-distance arcs can locate a perpendicular bisector

; it is not a guessed midpoint from a sketch. Coordinate evidence can support a proof, but one specially chosen picture does not establish a general statement for every triangle.

Rigid motions preserve lengths and angles. A 90° -degree anticlockwise rotation sends (x,y) to $(-y,x)$ about the origin. Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Equal angles establish similarity, but congruence also fixes size.



Original worked example from existing native teaching; transfer tasks use their own data.

Existing worked example: Point $(2, -1)$ translated by $(3, 4)$ becomes $(5, 3)$; rotated 90° counterclockwise about the origin it becomes $(1, 2)$. Between $(1, 2)$ and $(7, 10)$, distance is $\sqrt{6^2 + 8^2} = 10$ and midpoint $(4, 6)$. Two triangles with matching side lengths 3, 4, 5 are congruent by SSS; triangles with equal angles may instead differ by scale.

Complete original context

Every transfer question states all data it needs.

Independent practice and checked reasoning

■ Transfer 1

Rotate $A = (2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$. Reverse the order and compare endpoints.

Reasoning: Rotate first to $(3, 2)$, then translate to $(7, 3)$. Translate first to $(6, -2)$, then rotate to $(2, 6)$. The endpoints differ, so transformation order matters.

■ Transfer 2

$A = (-1, 2)$ and $B = (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation.

Reasoning: $d = \sqrt{(5 + 1)^2 + (10 - 2)^2} = 10$. Midpoint $M = ((-1 + 5)/2, (2 + 10)/2) = (2, 6)$. Segment slope is $8/6 = 4/3$, so perpendicular slope is $-3/4$. The bisector is

$$y - 6 = -(3/4)(x - 2).$$

■ Transfer 3

Two triangles have angles $40^\circ, 60^\circ, 80^\circ$. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas.

Reasoning: The triangles are similar by equal angles, not congruent because corresponding longest sides differ. Length ratio is $15/9 = 5/3$ and larger-to-smaller area ratio is $25/9$.

■ Transfer 4

Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3, 4, 5 cm.

Reasoning: Use the same compass radius, greater than half AB, to draw arcs centred at A and B meeting at P and Q on opposite sides. Draw PQ. $PA=PB$ and $QA=QB$ by equal radii. Thus both P and Q lie on the locus equidistant from A and B; PQ is the perpendicular bisector. Equivalently use congruent triangles and the common chord to establish perpendicularity and bisection. A ruler reading on one picture does not prove the general locus. SSS proves the two stated triangles congruent, fixing size as well as shape; Pythagoras also shows these particular triangles are right-angled.

■ Transfer 5

Two parallel lines are cut by a transversal. One corresponding angle is 68° . Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step.

Reasoning: Parallelism gives the matching corresponding angle 68° . The adjacent pair forms a straight angle, so the other is $180^\circ - 68^\circ = 112^\circ$. Equal corresponding angles require the stated parallel lines; the straight-angle sum uses adjacency on one line. A similar-looking sketch without parallelism would not justify the first equality.

Limits and next use

Follow the specified centre and order of transformations. Do not use a visually parallel or perpendicular pair as a stated condition.

All tasks here are public original practice with authored guidance. They are not official questions or fresh diagnostics. Existing protected tests and mocks remain separate.