

Real numbers, rational powers and complex arithmetic

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Scope and prerequisites

ACT framework, February 2026 revision. Original classroom cases are not an official form; preserve the scored/field-test boundaries of each source form.

- Distinguish rational and irrational numbers without assuming their sums retain type
- Apply exponent and radical rules on the permitted domain
- Calculate with complex numbers using $i^2 = -1$ and conjugates

Prerequisites: Distributive law; principal roots; $i^2 = -1$.

Explain and choose the method

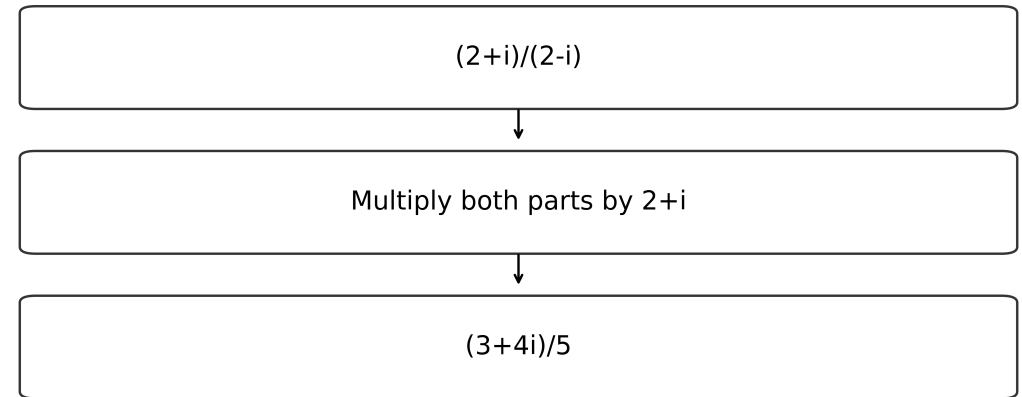
A rational number is a ratio of integers with nonzero denominator; a terminating or repeating decimal is rational. An irrational real number is not such a ratio. Closure of rational numbers under addition and multiplication does not imply closure of irrational numbers. Exact roots may simplify: $\sqrt{18} = 3\sqrt{2}$ remains irrational, whereas $\sqrt{16} = 4$ is rational.

For a positive base, $a^{m/n}$ combines an n th root and an integer power. A negative exponent forms a reciprocal and requires a nonzero base. Distinguish $\sqrt[n]{(x^2)} = |x|$ from x . When an even root is involved, respect the real domain and the principal nonnegative root. Distribute powers across products, not across sums: $(a+b)^2$ includes the cross term.

Complex arithmetic uses $i^2 = -1$. Add real parts and imaginary parts separately, and multiply by distribution before reducing i^2 . To divide by $a+bi$, multiply numerator and denominator by $a-bi$; the resulting denominator is a^2+b^2 when the original is nonzero. Complex roots permit solutions to equations such as $x^2 = -9$ that have no real roots.

Enhanced ACT still includes selected advanced topics, and no formula sheet is supplied. Build familiarity with the defining rules rather than guessing from a calculator display. Check whether the question asks for real or complex solutions, an exact radical or a decimal approximation. The local checks here are formative, not an official scored form.

A complex conjugate changes the imaginary sign. $(a+bi)(a-bi) = a^2 + b^2$ for real a, b . Thus $(2+i)/(2-i) = (2+i)^2/5 = (3+4i)/5$. The denominator is nonzero; distinguish an exact value from its decimal approximation.



Original diagram of the worked relationship; read the full wording and qualifications.

Existing worked example: $16^{3/4} = 2^3 = 8$. $(2+3i)(1-2i) = 2-4i+3i-6i^2 = 8-i$. For $(1+i)/(1-i)$, multiplying by $1+i$ gives $(1+i)^2/2 = 2i/2 = i$. The solutions of $x^2+9=0$ are $\pm 3i$; neither is real.

Complete original context

Every transfer question states all data it needs.

Independent practice and checked reasoning

■ Transfer 1

Calculate $(3-2i)/(1+i)$ in form $a+bi$.

Reasoning: Multiply both parts by $1-i$. Numerator is $(3-2i)(1-i) = 3-5i+2i^2 = 1-5i$. Denominator is 2. The answer is $1/2 - (5/2)i$. Multiplying back by $1+i$ returns $3-2i$.

■ Transfer 2

Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers.

Reasoning: The fifth root of 32 is 2, so $32^{2/5} = 2^2 = 4$. The principal square root is $|-7| = 7$. For the equation, $z^2 = -16$ gives $z = 4i, -4i$, both nonreal.

■ Transfer 3

Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational.

Reasoning: $\sqrt{2}\sqrt{2} = 2$ is rational. $\sqrt{2}\sqrt{3} = \sqrt{6}$ is irrational. Neither closure nor nonclosure for all products follows from the label irrational alone.

■ Transfer 4

Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3} . Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational.

Reasoning: $\sqrt{50} = 5\sqrt{2}$ is irrational, while $\sqrt{50}\sqrt{2} = \sqrt{100} = 10$ is rational. $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$ and $2^{-3} = 1/2^3 = 1/8$. The two irrational numbers sum to zero, which is rational; irrational numbers are not closed under addition. These even-root products use nonnegative real radicands.

Limits and next use

Do not infer that two irrational terms must have an irrational sum, distribute a square over addition, or replace $\sqrt{(x^2)}$ by x when x can be negative.

All tasks here are public original practice with authored guidance. They are not official questions or fresh diagnostics. Existing protected tests and mocks remain separate.