

Exercise walk-through

7

N-real-complex · Real numbers, rational powers and complex arithmetic

ACT · ACT · ACT

7 Real numbers, rational powers and complex arithmetic

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS complex conjugate • conjugate

Objective

- Distinguish rational and irrational numbers without assuming their sums retain type
- Apply exponent and radical rules on the permitted domain
- Calculate with complex numbers using $i^2 = -1$ and conjugates

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A rational number is a ratio of integers with nonzero denominator; a terminating or repeating decimal is rational. An irrational real number is not such a ratio. Closure of rational numbers under addition and multiplication does not imply closure of irra-

tional numbers. Exact roots may simplify: $\sqrt{18}=3\sqrt{2}$ remains irrational, whereas $\sqrt{16}=4$ is rational.

For a positive base, $a^{m/n}$ combines an n th root and an integer power. A negative exponent forms a reciprocal and requires a nonzero base. Distinguish $\sqrt{x^2}=|x|$ from x . When an even root is involved, respect the real domain and the principal nonnegative root. Distribute powers across products, not across sums: $(a+b)^2$ includes the cross term.

Complex arithmetic uses $i^2=-1$. Add real parts and imaginary parts separately, and multiply by distribution before reducing i^2 . To divide by $a+bi$, multiply numerator and denominator by $a-bi$; the resulting denominator is a^2+b^2 when the original is nonzero. Complex roots permit solutions to equations such as $x^2=-9$ that have no real roots.

Enhanced ACT still includes selected advanced topics, and no formula sheet is supplied. Build familiarity with the defining rules rather than guessing from a calculator display. Check whether the question asks for real or complex solutions, an exact radical or a

decimal approximation. The local checks here are formative, not an official scored form.

A complex conjugate $a-bi$ changes the imaginary sign. $(a+bi)(a-bi) = a^2+b^2$ for real a, b . Thus $(2+i)/(2-i) = (2+i)^2/5 = (3+4i)/5$. The denominator is nonzero; distinguish an exact value from its decimal approximation.

$$(2+i)/(2-i)$$



Multiply both parts by $2+i$



$$(3+4i)/5$$

Original diagram of the worked relationship; read the full wording and qualifications.

Worked example from the native lesson: $16^{3/4}=2^3=8$. $(2+3i)(1-2i)=2-4i+3i-6i^2=8-i$. For $(1+i)/(1-i)$, multiplying by $1+i$ gives $(1+i)^2/2=2i/2=i$. The solutions of $x^2+9=0$ are $\pm 3i$; neither is real.

Exercise 2.1

2.1 What is $(3+i)(3-i)$?

[1]

- A $9-i$
- B 6
- C 10
- D 8

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2.1 C – Conjugate product is $3^2+1^2=10$. A fails to cancel the imaginary cross terms; B subtracts rather than multiplying; D treats i^2 as $+1$ instead of -1 .

Correct response linked to the given case.

Exercises 2.2·2.3

2.2 $81^{(1/2)}$ equals:

[1]

- A 40.5
- B -9
- C 3
- D 9

2.3 The real part of $(4+2i)+(1-5i)$ is _____.

[1]

Exercises 2.2·2.3

2.2 $81^{(1/2)}$ equals:

[1]

- A 40.5
- B -9
- C 3
- D 9

2.2 D – The principal square root is 9. A halves the base instead of taking its root; B chooses the negative root rather than principal value; C takes a fourth root.

Correct response linked to the given case.

2.3 The real part of $(4+2i)+(1-5i)$ is _____.

[1]

2.3 5 – Real parts add to $4+1$; imaginary part is $-3i$.

Correct response linked to the given case.

Exercises 2.4·3.2

2.4 The sum of two irrational numbers is always irrational. [1]

3.1 Calculate $(3 - 2i)/(1 + i)$ in form $a + bi$. [3]

3.2 Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers. [3]

Exercises 2.4·3.2

2.4 The sum of two irrational numbers is always irrational. [1]

2.4 False – A number plus its negative gives rational zero.

Correct response linked to the given case.

3.1 Calculate $(3 - 2i)/(1 + i)$ in form $a + bi$. [3]

3.1 Multiply both parts by $1 - i$. Numerator is $(3 - 2i)(1 - i) = 3 - 5i + 2i^2 = 1 - 5i$. Denominator is 2. The answer is $1/2 - (5/2)i$. Multiplying back by $1 + i$ returns $3 - 2i$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers. [3]

3.2 The fifth root of 32 is 2, so $32^{2/5} = 2^2 = 4$. The principal square root is $|-7| = 7$. For the equation, $z^2 = -16$ gives $z = 4i, -4i$, both nonreal.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

Exercises 3.3·3.4

3.3 Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational. [3]

3.4 Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3} . Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational. [3]

Exercises 3.3·3.4

3.3 Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational. [3]

3.3 $\sqrt{2}\sqrt{2} = 2$ is rational. $\sqrt{2}\sqrt{3} = \sqrt{6}$ is irrational. Neither closure nor nonclosure for all products follows from the label irrational alone.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.4 Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3} . Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational. [3]

3.4 $\sqrt{50} = 5\sqrt{2}$ is irrational, while $\sqrt{50}\sqrt{2} = \sqrt{100} = 10$ is rational. $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$ and $2^{-3} = 1/2^3 = 1/8$. The two irrational numbers sum to zero, which is rational; irrational numbers are not closed under addition. These even-root products use nonnegative real radicands.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.