

Exercise walk-through

14

G-plane · Plane transformations, coordinate reasoning and proof

ACT · ACT · ACT

14 Plane transformations, coordinate reasoning and proof

Name: _____ Class: _____ Date: _____

Total: 19 marks

- KEY WORDS** perpendicular bisector • rigid motions • translation
- similarity • congruence • evidence • argument • transformation

Objective

- Transform coordinates while distinguishing rigid motions from dilation
- Use distance, midpoint and slope as geometric evidence
- Use valid congruence and angle relationships rather than a diagram's appearance

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A translation adds a displacement to each point; reflection across the x-axis sends (x,y) to $(x,-y)$. A 90° counterclockwise rotation around the origin sends (x,y) to $(-y,x)$. These rigid motions preserve lengths and angles. Dilation by factor k about the origin

scales coordinates and lengths by k when k is positive, preserving shape but generally changing size.

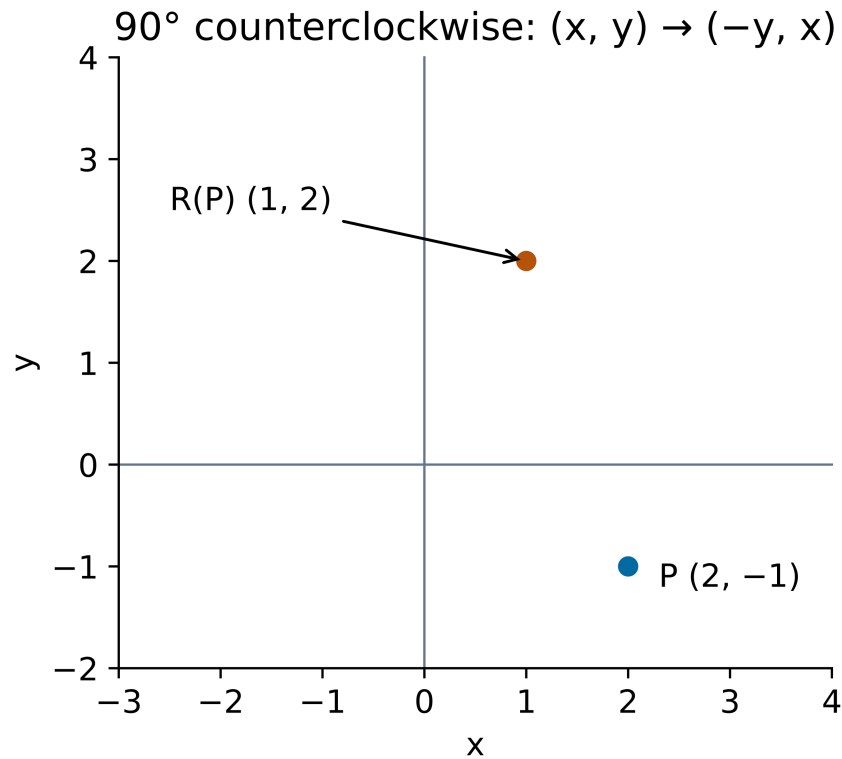
Distance between two points follows the Pythagorean theorem: square the coordinate differences, add and take the root. The midpoint averages each coordinate. Slope compares vertical and horizontal change; perpendicular nonvertical slopes have product -1 . Vertical and horizontal lines require separate treatment rather than division by zero.

A geometric argument starts from stated or marked conditions. Vertical angles are equal; parallel lines give corresponding and alternate-angle relationships. Triangle angles sum to 180° . SSS, SAS and ASA/AAS can establish congruence with proper correspondence, while two equal angles establish similarity but not equal size.

Construction reasoning uses a compass to transfer lengths and draw equal-radius arcs. An intersection of equal-distance arcs can locate a perpendicular bisector; it is not a guessed midpoint from a sketch. Coordinate evidence can support a proof, but

one specially chosen picture does not establish a general statement for every triangle.

Rigid motions preserve lengths and angles. A 90-degree anticlockwise rotation sends (x, y) to $(-y, x)$ about the origin. Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Equal angles establish similarity , but congruence also fixes size.



Original worked example from existing native teaching; transfer tasks use their own data.

Worked example from the native lesson: Point $(2, -1)$ translated by $(3, 4)$ becomes $(5, 3)$; rotated 90° counterclockwise about the origin it becomes $(1, 2)$. Between $(1, 2)$ and $(7, 10)$, distance is $\sqrt{6^2 + 8^2} = 10$ and midpoint $(4, 6)$. Two triangles with matching side lengths 3, 4, 5 are congruent by SSS; triangles with equal angles may instead differ by scale.

Exercise 2.1

2.1 Rotate $(3,2)$ 90° counterclockwise about the origin.

[1]

- **A** $(3,-2)$
- **B** $(-2,3)$
- **C** $(2,-3)$
- **D** $(-3,2)$

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- **A** $(3,-2)$
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- **D** $(-3,2)$

2.1 B – Apply $(-y,x)$. A reflects in the horizontal axis; C rotates clockwise; D reflects in the vertical axis. None is the stated anticlockwise quarter-turn.

Correct response linked to the given case.

Exercises 2.2-2.3

2.2 Midpoint of $(0,2)$ and $(6,8)$:

[1]

- **A** $(3,3)$
- **B** $(0,8)$
- **C** $(3,5)$
- **D** $(6,10)$

2.3 A line of slope 2 has a perpendicular nonvertical slope _____.

[1]

Exercises 2.2·2.3

2.2 Midpoint of (0,2) and (6,8): [1]

- A (3,3)
- B (0,8)
- C (3,5)
- D (6,10)

2.2 C – Average corresponding coordinates. A averages the y difference rather than y coordinates; B combines endpoints without averaging; D adds coordinates without halving.

Correct response linked to the given case.

2.3 A line of slope 2 has a perpendicular nonvertical slope _____. [1]

2.3 $-1/2$ – The slopes multiply to -1.

Correct response linked to the given case.

Exercises 2.4·3.1

2.4 Two equal triangle angles prove congruence.

[1]

3.1 Rotate $A = (2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$. Reverse the order and compare endpoints.

[3]

Exercises 2.4·3.1

2.4 Two equal triangle angles prove congruence. [1]

2.4 False – They establish similarity; lengths may differ.

Correct response linked to the given case.

3.1 Rotate $A = (2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$. Reverse the order and compare endpoints. [3]

3.1 Rotate first to $(3, 2)$, then translate to $(7, 3)$. Translate first to $(6, -2)$, then rotate to $(2, 6)$. The endpoints differ, so transformation order matters.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

Exercises 3.2-3.3

3.2 $A = (-1, 2)$ and $B = (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation. [3]

3.3 Two triangles have angles $40^\circ, 60^\circ, 80^\circ$. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas. [3]

Exercises 3.2·3.3

3.2 $A = (-1, 2)$ and $B = (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation. [3]

3.2 $d = \sqrt{(5+1)^2 + (10-2)^2} = 10$. Midpoint $M = ((-1+5)/2, (2+10)/2) = (2, 6)$. Segment slope is $8/6 = 4/3$, so perpendicular slope is $-3/4$. The bisector is $y - 6 = -(3/4)(x - 2)$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 Two triangles have angles $40^\circ, 60^\circ, 80^\circ$. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas. [3]

3.3 The triangles are similar by equal angles, not congruent because corresponding longest sides differ. Length ratio is $15/9 = 5/3$ and larger-to-smaller area ratio is $25/9$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

Exercise 3.4

3.4 Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3,4,5 cm. [3]

Exercise 3.4

3.4 Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3,4,5 cm. [3]

3.4 Use the same compass radius, greater than half AB, to draw arcs centred at A and B meeting at P and Q on opposite sides. Draw PQ. $PA=PB$ and $QA=QB$ by equal radii. Thus both P and Q lie on the locus equidistant from A and B; PQ is the perpendicular bisector. Equivalently use congruent triangles and the common chord to establish perpendicularity and bisection. A ruler reading on one picture does not prove the general locus. SSS proves the two stated triangles congruent, fixing size as well as shape; Pythagoras also shows these particular triangles are right-angled.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

Exercise 3.5

3.5 Two parallel lines are cut by a transversal. One corresponding angle is 68° . Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step. [3]

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3.5 Parallelism gives the matching corresponding angle 68° . The adjacent pair forms a straight angle, so the other is $180^\circ - 68^\circ = 112^\circ$. Equal corresponding angles require the

stated parallel lines; the straight-angle sum uses adjacency on one line. A similar-looking sketch without parallelism would not justify the first equality.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.