

Exercise walk-through

11

F-compose · Functions, composition, transformations and inverses

ACT · ACT · ACT

11 Functions, composition, transformations and inverses

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS transformation • composition • one-to-one

Objective

- Evaluate and convert between function rules, tables and graphs
- Track input order and domain through composition
- Find an inverse after choosing a one-to-one domain

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A function gives one output for each allowed input. A graph passing a vertical line at two different output points is not a function of x . Domain lists allowed inputs; range lists attained outputs. Convert a table to a rule only when the relationship is supported, and

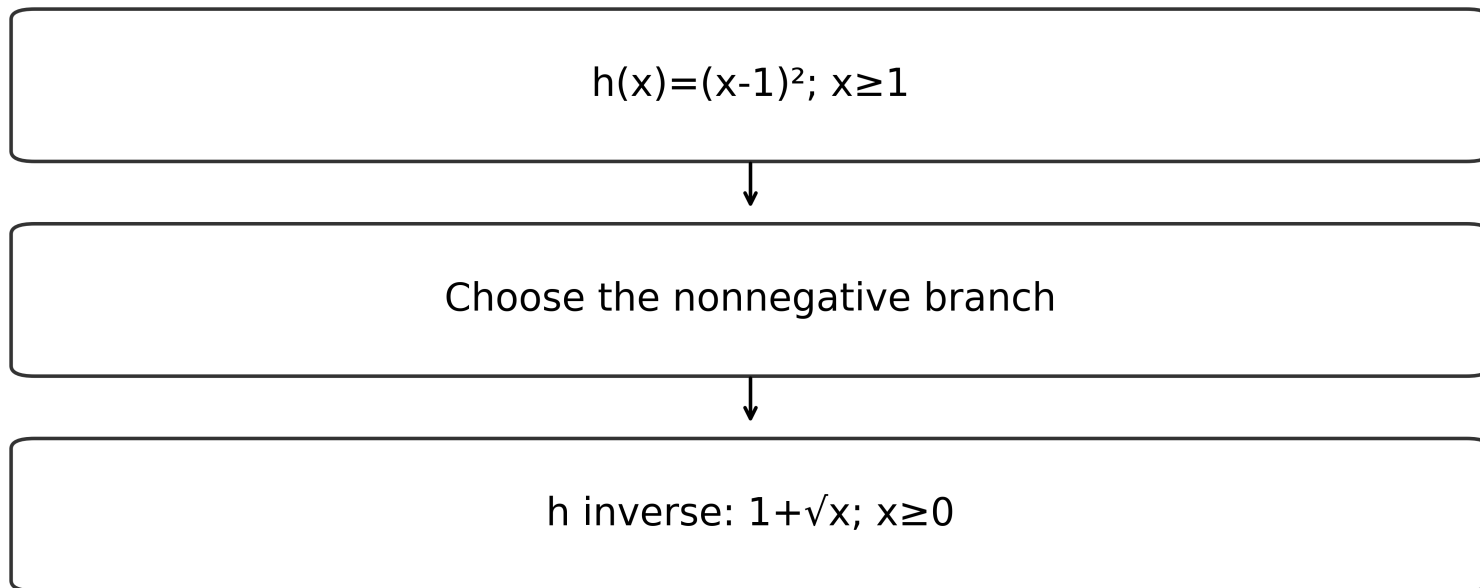
interpret each variable's role.

In $f(g(x))$, the inner function g acts first and its output must be in f 's domain. For $f(u)=1/(u-1)$ and $g(x)=x^2+1$, composition gives $1/x^2$ and excludes $x=0$. Domain checking belongs to the original functions as well as the simplified result.

An output change $f(x)+k$ shifts vertically by k ; $f(x-h)$ shifts right by h . An outside multiplier changes output scale, while an inside multiplier changes how quickly the input reaches old points. Identify a known point to verify the transformation instead of guessing direction from a sign.

An inverse exchanges input and output and requires a one-to-one relationship on the chosen domain. Solve $y=f(x)$ for x , then exchange names. A quadratic must usually be restricted to one side of its vertex. The inverse function f^{-1} is not the reciprocal $1/f$. Check composition in both directions on the allowed domain.

Composition applies the inside rule first. If $f(u) = 1/(u-2)$ and $g(x) = x^2+2$, $f(g(x)) = 1/x^2$, with $x \neq 0$. An inverse reverses input and output on a one-to-one domain. Restrict $h(x) = (x-1)^2$ to $x \geq 1$ before taking $h^{-1}(x) = 1 + \sqrt{x}$.



Original diagram of the worked relationship; read the full wording and qualifications.

Worked example from the native lesson: $f(x)=2x+3$ and $g(x)=x^2$ give $f(g(x))=2x^2+3$, but $g(f(x))=(2x+3)^2$. For $h(x)=(x-2)^2$ restricted to $x \geq 2$, the inverse is $2+\sqrt{x}$ with domain $x \geq 0$. A vertical shift of h by 4 has vertex $(2,4)$; the unrestricted quadratic would not have a single-valued inverse.

Exercise 2.1

2.1 $f(x)=3x+1$, $g(x)=x-2$. $f(g(5))$ equals:

[1]

- **A** 7
- **B** 16
- **C** 10
- **D** 14

Exercise 2.1

2.1 $f(x)=3x+1$, $g(x)=x-2$. $f(g(5))$ equals:

[1]

- **A** 7
- **B** 16
- **C** 10
- **D** 14

2.1 C – $g(5)=3$, then $f(3)=10$. A has no correct inside-then-outside substitution; B stops at $f(5)$; D computes $g(f(5))$, reversing the order.

Correct response linked to the given case.

Exercises 2.2·2.3

2.2 The inverse of $f(x)=4x-8$ is:

[1]

- **A** $1/(4x-8)$
- **B** $4x+8$
- **C** $(x-8)/4$
- **D** $(x+8)/4$

2.3 The vertex of $(x-5)^2+2$ has x-coordinate _____.

[1]

Exercises 2.2·2.3

2.2 The inverse of $f(x)=4x-8$ is: [1]

- A $1/(4x-8)$
- B $4x+8$
- C $(x-8)/4$
- D $(x+8)/4$

2.2 D – Solve $y=4x-8$ for $x=(y+8)/4$. A is a reciprocal rather than an inverse; B does not undo multiplication by 4; C subtracts 8 where solving requires adding it.

Correct response linked to the given case.

2.3 The vertex of $(x-5)^2+2$ has x-coordinate _____. [1]

2.3 5 – The horizontal shift is right by 5.

Correct response linked to the given case.

Exercises 2.4·3.2

2.4 An unrestricted quadratic has a single-valued inverse on all real outputs. [1]

3.1 Let $f(u) = 1/(u - 3)$ and $g(x) = x^2 + 3$. Find $f(g(x))$ with domain. Find $g(f(4))$

3.2 Restrict $h(x) = (x + 2)^2$ to $x \leq -2$. Find its inverse, including domain and range.

Exercises 2.4·3.2

2.4 An unrestricted quadratic has a single-valued inverse on all real outputs. [1]

2.4 False – Two different inputs can share an output; a branch restriction is needed.

Correct response linked to the given case.

3.1 Let $f(u) = 1/(u - 3)$ and $g(x) = x^2 + 3$. Find $f(g(x))$ with domain. Find $g(f(4))$

3.1 $f(g(x)) = 1/(x^2 + 3 - 3) = 1/x^2$, excluding $x = 0$. For the other order, $f(4) = 1$ and $g(1) = 4$. Composition order changes both operation and possibly domain; $f(g(4)) = 1/16$ is different.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Restrict $h(x) = (x + 2)^2$ to $x \leq -2$. Find its inverse, including domain and range.

3.2 Solve $y = (x + 2)^2$ with $x + 2 \leq 0$ to get $x = -2 - \sqrt{y}$. Thus $h^{-1}(x) = -2 - \sqrt{x}$, domain $x \geq 0$, range $y \leq -2$. The plus branch returns values outside the chosen original domain.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

Exercise 3.3

3.3 The graph of $f(x) = x^2$ is changed to $g(x) = 3f(x + 4) - 2$. Find its vertex and

Exercise 3.3

3.3 The graph of $f(x) = x^2$ is changed to $g(x) = 3f(x + 4) - 2$. Find its vertex and

3.3 $g(x) = 3(x + 4)^2 - 2$ has vertex $(-4, -2)$ and range $y \geq -2$. Old point $(1, 1)$ maps to $(-3, 1)$ because $3(1) - 2 = 1$; substitution confirms $g(-3) = 1$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.