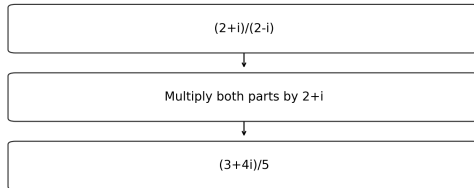


Real numbers, rational powers and complex arithmetic

Original skill preparation; versioned source scope

Method and conditions



- A complex conjugate changes the imaginary sign.
- Distributive law; principal roots; $i^2 = -1$.

Worked method 1

Read and reason

A complex conjugate changes the imaginary sign. $(a + bi)(a - bi) = a^2 + b^2$ for real a, b . Thus $(2 + i)/(2 - i) = (2 + i)^2/5 = (3 + 4i)/5$. The denominator is nonzero; distinguish an exact value from its decimal approximation.

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

Calculate $(3 - 2i)/(1 + i)$ in form $a + bi$.

Reasoning 1.1

Read and reason

Multiply both parts by $1 - i$. Numerator is $(3 - 2i)(1 - i) = 3 - 5i + 2i^2 = 1 - 5i$.
Denominator is 2. The answer is $1/2 - (5/2)i$. Multiplying back by $1 + i$ returns $3 - 2i$.

Attempt 2.1

Read and reason

Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers.

Reasoning 2.1

Read and reason

The fifth root of 32 is 2, so $32^{2/5} = 2^2 = 4$. The principal square root is $|-7| = 7$. For the equation, $z^2 = -16$ gives $z = 4i, -4i$, both nonreal.

Attempt 3.1

Read and reason

Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational.

Reasoning 3.1

Read and reason

$\sqrt{2}\sqrt{2} = 2$ is rational. $\sqrt{2}\sqrt{3} = \sqrt{6}$ is irrational. Neither closure nor nonclosure for all products follows from the label irrational alone.

Attempt 4.1

Read and reason

Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3} . Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational.

Reasoning 4.1

Read and reason

$\sqrt{50} = 5\sqrt{2}$ is irrational, while $\sqrt{50}\sqrt{2} = \sqrt{100} = 10$ is rational.

$32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$ and $2^{-3} = 1/2^3 = 1/8$. The two irrational numbers sum to zero, which is rational; irrational numbers are not closed under addition. These even-root products use nonnegative real radicands.

Transfer and limits

Read and reason

Distributive law; principal roots; $i^2 = -1$.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.