7 Real numbers, rational powers and complex arithmetic

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS complex conjugate • conjugate

Objective

- Distinguish rational and irrational numbers without assuming their sums retain type
- Apply exponent and radical rules on the permitted domain
- Calculate with complex numbers using $i^2=-1$ and conjugates

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

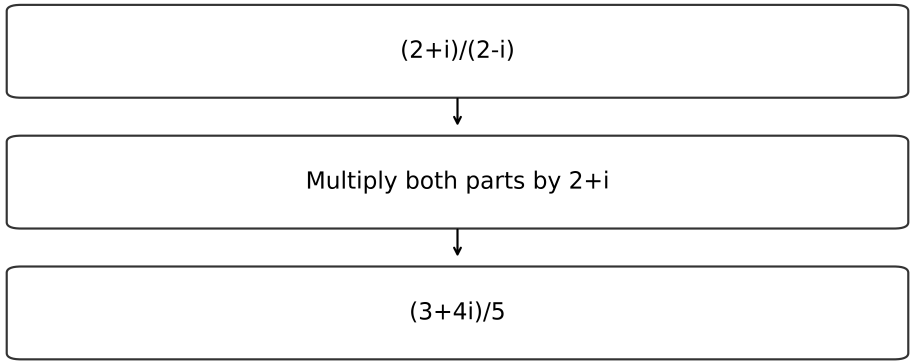
A rational number is a ratio of integers with nonzero denominator; a terminating or repeating decimal is rational. An irrational real number is not such a ratio. Closure of rational numbers under addition and multiplication does not imply closure of irrational numbers. Exact roots may simplify: $\sqrt{18}=3\sqrt{2}$ remains irrational, whereas $\sqrt{16}=4$ is rational.

For a positive base, a^(m/n) combines an nth root and an integer power. A negative exponent forms a reciprocal and requires a nonzero base. Distinguish $\sqrt{(x^2)}=|x|$ from x. When an even root is involved, respect the real domain and the principal nonnegative root. Distribute powers across products, not across sums: $(a+b)^2$ includes the cross term.

Complex arithmetic uses $i^2=-1$. Add real parts and imaginary parts separately, and multiply by distribution before reducing i^2. To divide by a+bi, multiply numerator and denominator by a-bi; the resulting denominator is a^2+b^2 when the original is nonzero. Complex roots permit solutions to equations such as $x^2=-9$ that have no real roots.

Enhanced ACT still includes selected advanced topics, and no formula sheet is supplied. Build familiarity with the defining rules rather than guessing from a calculator display. Check whether the question asks for real or complex solutions, an exact radical or a decimal approximation. The local checks here are formative, not an official scored form.

A complex conjugate changes the imaginary sign. $(a+bi)(a-bi)=a^2+b^2$ for real a, b. Thus $(2+i)/(2-i)=(2+i)^2/5=(3+4i)/5$. The denominator is nonzero; distinguish an exact value from its decimal approximation.



Worked steps: (2+i)/(2-i) → Multiply both parts by 2+i → (3+4i)/5

Worked example. 16^(3/4)=2^3=8. (2+3i)(1-2i)=2-4i+3i-$6i^2$=8-i. For (1+i)/(1-i), multiplying by 1+i gives $(1+i)^2/2=2i/2=i$. The solutions of $x^2+9=0$ are $\pm 3i$; neither is real.

2 Practice

2.1 What is (3+i)(3-i)? [1]

- **A** 9-i
- **B** 6
- **C** 10
- **D** 8

2.2 81^(1/2) equals: [1]

- **A** 40.5
- **B** -9
- **C** 3
- **D** 9

2.3 The real part of (4+2i)+(1-5i) is _____. [1]

2.4 The sum of two irrational numbers is always irrational. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Calculate $(3 - 2i)/(1 + i)$ in form $a + bi$. [3]

3.2 Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers. [3]

3.3 Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational. [3]

3.4 Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3}. Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational. [3]