

17 Sampling inference, expected value and counting

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS conditional probability • margin of error
 • expected value • inference • evidence

Objective

- Interpret a population estimate and simulation evidence without claiming proof
- Calculate conditional, compound and expected-value quantities
- Choose permutations or combinations according to whether order matters

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A random sample supports inference to its source population; a margin of error gives a sampling-uncertainty interval around an estimate. Random assignment instead supports a causal treatment comparison. In a simulation, a result rarely produced under a claim can be evidence against that claim; failure to find an unusual result does not prove the claim true.

Conditional probability restricts the denominator to the conditioning group. For either A or B, subtract their overlap after adding separate probabilities. Independence permits multiplication of unchanged probabilities; without-replacement draws require updated counts. Expected value is the sum of each possible outcome times its probability, not necessarily an outcome achieved in one trial.

Use the multiplication principle for successive choices. For n distinct objects, choosing r in order gives $n!/(n-r)!$; choosing an unranked set gives $n!/r!(n-r)!$. Dividing by $r!$ removes the multiple orderings of the same set. These formulas assume distinct objects and no replacement; different conditions need a corresponding count.

Translate the event to a count before calculating its probability. If a favourable event can occur in several disjoint ways, add those ways; if paths overlap, avoid double-counting. A numerical answer must be between zero and one for a probability, while an expected cost or score has the units of the outcomes.

A margin of error is added and subtracted in percentage points. Estimate 63% with margin 4 points gives 59%–67%. It is an interval from a procedure, not a guarantee that each person's response is near 63%. Random sampling supports population inference; random assignment addresses treatment comparisons.

Choose 2 of 4: six unordered pairs; AB and BA count once

AB	AC	AD
BC	BD	CD

Choosing 2 of 4 gives six pairs, because AB and BA are the same pair

Worked example. From five students, captain/deputy pairs number $5 \cdot 4 = 20$; unranked pairs number $5 \cdot 4/2 = 10$. A game awards 4 points with probability $1/4$ and 0 otherwise, so expected award is 1 point per play. A sample estimate $52\% \pm 3$ percentage points gives 49%–55%; it is not a guarantee about every sample or individual.

2 Practice

2.1 Choose two unranked representatives from four people. Number of choices: [1]

- A 6
- B 12
- C 8
- D 16

2.2 A random award is 10 with probability 0.2 and 0 otherwise. Expected award: [1]

- A 0.2
- B 2
- C 10
- D 5

2.3 An estimate $64\% \pm 4$ points has lower endpoint _____. [1]

2.4 Failing to reject a claim establishes that it is certainly true. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Six students choose a chairperson and a different secretary. How many assignments? How many unranked two-person committees? Explain the difference. [3]

3.2 A game pays 12 tokens with probability 0.1, 3 with probability 0.4, and zero otherwise. Entry costs 2 tokens. Find expected payout and expected net gain. Does a player receive that amount every time? [3]

3.3 Two balls are drawn without replacement from a bag with 3 red and 2 blue balls. Find the probability both are red and of exactly one red. [3]

3.4 A random sample of 500 registered library members reports 62% support for a change, with stated margin 4 percentage points. State the interval and population. Does it establish support among every town resident? [3]

3.5 Forty volunteers are randomly assigned to old or new instructions, with the same task and conditions. The new group finishes faster. What causal and population claims are reasonable? [3]

3.6 A poll of 10000 website volunteers has a narrow reported margin. Explain why size alone cannot repair selection bias, and propose a better recruitment method. [3]
