

14 Plane transformations, coordinate reasoning and proof

Name: _____ Class: _____ Date: _____

Total: 19 marks

KEY WORDS perpendicular bisector • rigid motions • translation
 • similarity • congruence • evidence • argument • transformation

Objective

- Transform coordinates while distinguishing rigid motions from dilation
- Use distance, midpoint and slope as geometric evidence
- Use valid congruence and angle relationships rather than a diagram's appearance

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

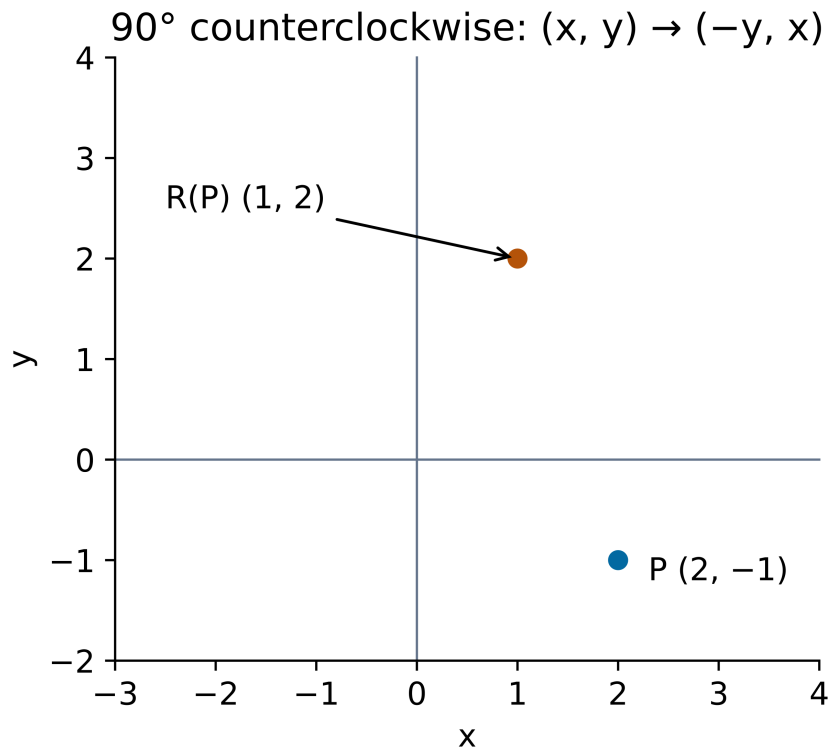
A translation adds a displacement to each point; reflection across the x-axis sends (x,y) to $(x,-y)$. A 90° counterclockwise rotation around the origin sends (x,y) to $(-y,x)$. These rigid motions preserve lengths and angles. Dilation by factor k about the origin scales coordinates and lengths by k when k is positive, preserving shape but generally changing size.

Distance between two points follows the Pythagorean theorem: square the coordinate differences, add and take the root. The midpoint averages each coordinate. Slope compares vertical and horizontal change; perpendicular nonvertical slopes have product -1 . Vertical and horizontal lines require separate treatment rather than division by zero.

A geometric argument starts from stated or marked conditions. Vertical angles are equal; parallel lines give corresponding and alternate-angle relationships. Triangle angles sum to 180° . SSS, SAS and ASA/AAS can establish congruence with proper correspondence, while two equal angles establish similarity but not equal size.

Construction reasoning uses a compass to transfer lengths and draw equal-radius arcs. An intersection of equal-distance arcs can locate a perpendicular bisector; it is not a guessed midpoint from a sketch. Coordinate evidence can support a proof, but one specially chosen picture does not establish a general statement for every triangle.

Rigid motions preserve lengths and angles. A 90° -degree anticlockwise rotation sends (x,y) to $(-y,x)$ about the origin. Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Equal angles establish similarity, but congruence also fixes size.



A 90° counterclockwise rotation sends (x, y) to $(-y, x)$

Worked example. Point $(2, -1)$ translated by $(3, 4)$ becomes $(5, 3)$; rotated 90° counterclockwise about the origin it becomes $(1, 2)$. Between $(1, 2)$ and $(7, 10)$, distance is $\sqrt{6^2 + 8^2} = 10$ and midpoint $(4, 6)$. Two triangles with matching side lengths 3, 4, 5 are congruent by SSS; triangles with equal angles may instead differ by scale.

2 Practice

2.1 Rotate $(3, 2)$ 90° counterclockwise about the origin.

[1]

- A $(3, -2)$
- B $(-2, 3)$
- C $(2, -3)$
- D $(-3, 2)$

2.2 Midpoint of $(0,2)$ and $(6,8)$: [1]

- **A** $(3,3)$
- **B** $(0,8)$
- **C** $(3,5)$
- **D** $(6,10)$

2.3 A line of slope 2 has a perpendicular nonvertical slope _____. [1]

2.4 Two equal triangle angles prove congruence. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Rotate $A = (2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$. Reverse the order and compare endpoints. [3]

3.2 $A = (-1, 2)$ and $B = (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation. [3]

3.3 Two triangles have angles $40^\circ, 60^\circ, 80^\circ$. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas. [3]

3.4 Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3, 4, 5 cm. [3]

3.5 Two parallel lines are cut by a transversal. One corresponding angle is 68° . Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step. [3]
