

13 Trigonometric graphs, identities and general triangles

Name: _____ Class: _____ Date: _____ Total: 16 marks

KEY WORDS included angle • cosine rule • amplitude

Objective

- Interpret amplitude, period and vertical shift in a sinusoidal rule
- Use unit-circle signs and basic identities
- Choose a sine, cosine or area relation for a general triangle

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

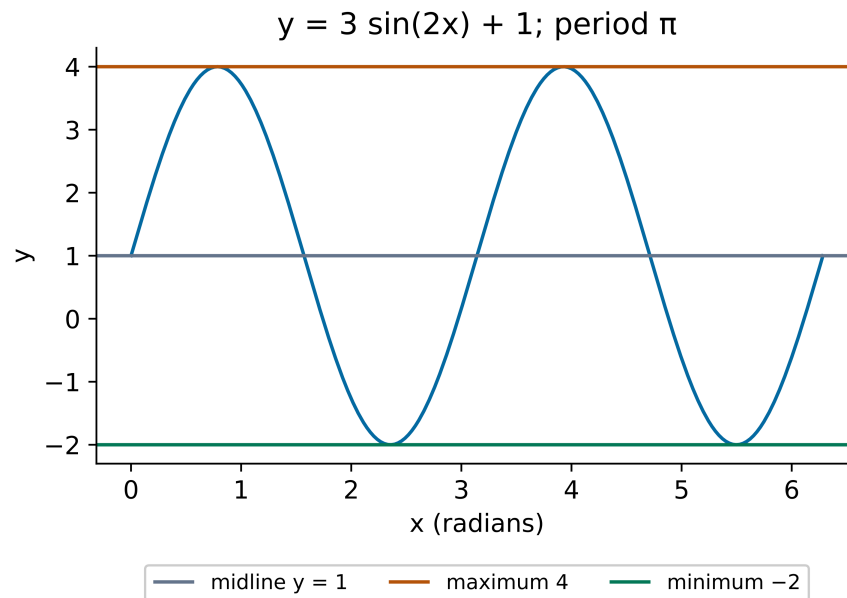
For a right triangle, sin is opposite/hypotenuse, cos adjacent/hypotenuse and tan opposite/adjacent relative to a chosen angle. Similar triangles explain why these ratios depend on angle rather than size. On the unit circle, coordinates are $(\cos \theta, \sin \theta)$, extending the ratios to other quadrants with appropriate signs.

$\sin^2 \theta + \cos^2 \theta = 1$ and $\tan \theta = \sin \theta / \cos \theta$ when $\cos \theta \neq 0$. For $y = A \sin(Bx) + D$ with radian x , amplitude is $|A|$, period $2\pi/|B|$ and midline D . A negative A reflects the wave; D shifts it vertically. Check whether the input is in radians or degrees before using a graph interval.

For a general triangle, the cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$ uses the included angle C between a and b . The sine rule compares each side with the sine of its opposite angle. Area is $ab \sin C / 2$ when two sides and their included angle are known. Choose the relationship matching the supplied information.

Do not infer a right angle from the sketch. A sine-based equation may allow more than one angle in a permitted interval; retain the quadrant information and any triangle sum condition. Keep amplitude distinct from total peak-to-trough distance, which is twice the amplitude.

Amplitude is half the peak-to-trough distance. In $y = A \sin(Bx) + D$, it is $|A|$; radian period is $2\pi/|B|$. For a non-right triangle, match known sides and included angle before selecting the cosine rule. $c^2 = a^2 + b^2 - 2ab \cos C$.



Amplitude 3 about the midline $y = 1$; the factor 2 makes the period π

Worked example. $y = 3 \sin(2x) + 1$ has amplitude 3, period π and values between -2 and 4. A triangle with sides 5 and 7 around a 60° angle has third side $\sqrt{(25+49-70 \cdot 1/2)} = \sqrt{39}$ and area $35\sqrt{3}/4$. At 150° , sine is $1/2$ and cosine $-\sqrt{3}/2$.

2 Practice

2.1 $y = 2 \sin(3x) + 4$ has maximum: [1]

- A 6
- B 2
- C 4
- D 8

2.2 For radian x , $\sin(2x)$ has period: [1]

- A $1/2$
- B π
- C 2π
- D 4π

2.3 $\sin^2\theta + \cos^2\theta =$ _____. [1]

2.4 The Pythagorean theorem directly applies to every triangle. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 For radian input $y = -4\sin(2x) + 3$, give amplitude, period, range and value at $x = \pi/4$. [3]

3.2 A triangle has sides 6 cm and 10 cm enclosing 60° . Find its third side and area without assuming it is right-angled. [3]

3.3 Find every angle in $0^\circ \leq \theta < 360^\circ$ with $\sin \theta = 1/2$. For each, give cosine and tangent. [3]

3.4 A right triangle has legs 9 and 12 cm. Find the hypotenuse and sine/cosine of the angle opposite 9 cm. A different triangle has angles 30° and 45° and side 8 cm opposite 30° . Find the side opposite 45° , explaining why the sine rule fits. [3]
