

12 Sequences, finite sums, exponentials and logarithms

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS arithmetic sequence • geometric sequence • logarithm
• argument

Objective

- Distinguish arithmetic difference from geometric ratio
- Find a term or finite sum while keeping indexing consistent
- Convert logarithmic equations to exponential form on the valid domain

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

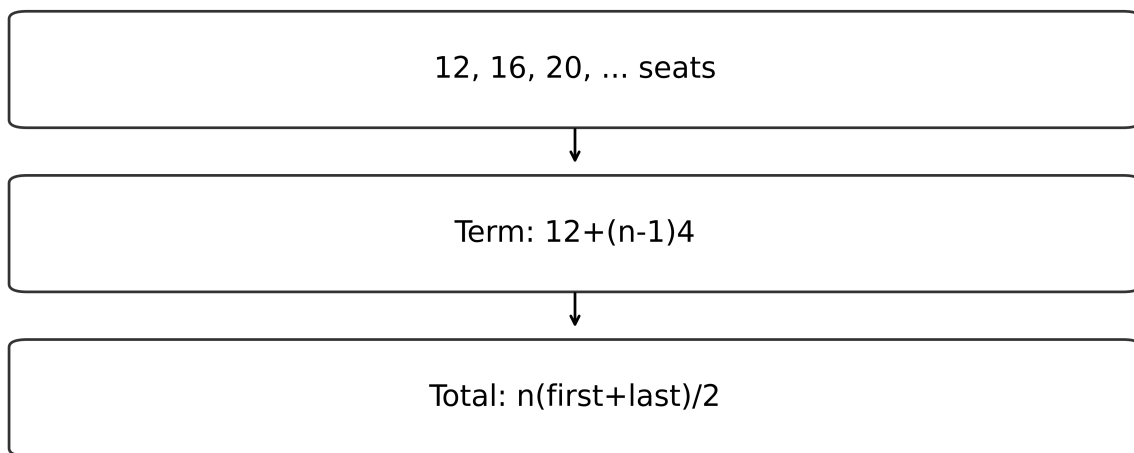
An arithmetic sequence has constant difference d : $a_n = a_1 + (n-1)d$. Its finite sum is $n(a_1 + a_n)/2$. A geometric sequence has constant ratio r : $a_n = a_1 r^{(n-1)}$. For $r \neq 1$, a finite sum is $a_1(1-r^n)/(1-r)$; when $r=1$ it is simply $n a_1$. Match the requested term or total.

An exponential model $A b^t$ represents multiplication by b per stated time unit. A percentage growth r uses factor $1+r$; repeated changes multiply factors rather than adding percentages. If doubling takes several time units, divide t by that interval in the exponent.

For base $b > 0$ with $b \neq 1$, $\log_b(a) = c$ means $b^c = a$ and requires $a > 0$. Solve a logarithmic equation by translating it to an exponential relationship, then check the argument. Logarithms of products add under valid positive arguments; logarithms do not distribute over addition.

Use exact powers when available. ACT may test a simple logarithmic relationship without requiring numerical logarithm tables. A calculated approximation must still respect the question's representation and domain. Keep a sequence's index, an exponential model's time and a logarithm's argument conceptually separate.

Arithmetic sequence : $a_n = a_1 + (n-1)d$. Its sum is $S_n = n(a_1 + a_n)/2$. Geometric sequence : $a_n = a_1 r^{n-1}$ and $S_n = a_1(1-r^n)/(1-r)$ for $r \neq 1$. A logarithm reverses exponentiation: $\log_b a = c$ means $b^c = a$, with $a > 0$, $b > 0$, $b \neq 1$.



Worked steps: 12, 16, 20, ... seats $\rightarrow$ Term: $12+(n-1)4 \rightarrow$ Total: $n(\text{first}+\text{last})/2$

Worked example. Arithmetic 4,7,10,... has $a_5=16$ and sum of first five= $5(4+16)/2=50$. Geometric 3,6,12,24 has four-term sum 45. $\log_2(x-1)=4$ gives $x-1=16$, so $x=17$, which satisfies $x>1$. A population $50 \cdot 2^{(t/3)}$ reaches 200 after six time units.

2 Practice

2.1 Arithmetic sequence starts at 5 with difference 4. Fourth term: [1]

- A 21
- B 9
- C 20
- D 17

2.2 $\log_3 81$ equals: [1]

- A 4
- B 3
- C 27
- D 9

2.3 The first three terms 2,6,18 have sum _____. [1]

2.4 $\log(a+b)=\log a+\log b$ for all positive a,b . [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Seats in successive rows number 12,16,20,... . Find the eighth row and the total in the first eight rows. State the model assumption. [3]

3.2 A geometric sequence starts 5,15,45,... . Find its fifth term and sum of the first five terms. Explain what changes if the ratio is 1. [3]

3.3 Solve $\log_2(x-3) = 5$ and explain why $\log_2(x+3)$ cannot generally be split into $\log_2 x + \log_2 3$. [3]

3.4 A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t , find $N(4)$, and distinguish factor from percentage growth. [3]
