

# 9 Linear, quadratic and polynomial relationships

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 13 marks**

**KEY WORDS** vertex form • parabola • rate

## Objective

- Choose equivalent expanded, factored or vertex forms for a task
- Solve linear and polynomial equations using structure
- Determine quadratic inequality regions from signs rather than roots alone

Original public practice; marks below are classroom self-check points, not official criteria.

## 1 Key knowledge and worked examples

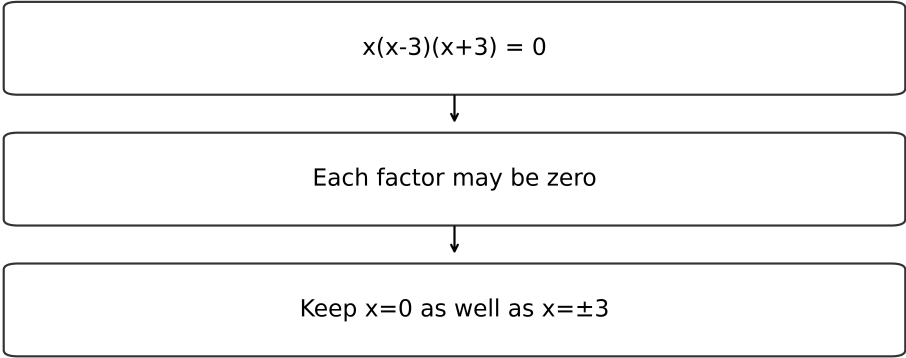
Translate relationships into expressions before manipulating them. A fixed fee plus a per-unit charge is an affine linear model, while a product of dimensions can create a quadratic. An equivalent form should preserve the original expression on its domain. Factor common terms and recognise identities before expanding every product.

A linear equation can have one, no or infinitely many solutions after simplifying; check zero coefficients before dividing. Multiplying or dividing an inequality by a negative reverses its direction. A polynomial product equals zero when at least one factor is zero. Keep all factors, including an extracted  $x$ , to avoid losing a zero root.

A quadratic's factored form exposes roots and its vertex form exposes an extremum. For an inequality, order the real roots and test the sign in each interval. Include boundaries only for  $\leq$  or  $\geq$ . A parabola opening upward is negative between two distinct real roots and positive outside them.

Use substitution into the original relation to check candidate values. An expanded coefficient question may require combining several contributions to the same power. A degree-three polynomial need not require a general cubic formula: a common factor or known root can reduce it to simpler factors. Choose the method based on structure.

An identity holds for every permitted input. Start with  $ax + b = cx + d$ . Collect terms:  $(a - c)x = d - b$ . Inspect  $a - c$  before dividing. If  $a = c$  and  $b = d$ , every real input works. If only  $a = c$ , no input works. Otherwise  $x = (d - b)/(a - c)$  gives one solution.



*Worked steps:*  $x(x-3)(x+3) = 0 \rightarrow$  Each factor may be zero  $\rightarrow$  Keep  $x=0$  as well as  $x=\pm 3$

**Worked example.**  $x^2-5x+6=(x-2)(x-3)$ , so  $x^2-5x+6<0$  gives  $2<x<3$ .  $x^3-4x=x(x-2)(x+2)$ , giving roots  $-2,0,2$ . The expression  $2(x-3)^2+5$  has minimum 5 at  $x=3$ . In  $(2x+1)(x-4)$ , the  $x$  coefficient is  $-8+1=-7$ .

## 2 Practice

**2.1**  $x(x-5)=0$  has solutions: [1]

- A 0 and 5
- B 5 only
- C 0 only
- D -5 and 0

**2.2**  $(x-1)(x-4)<0$  holds on: [1]

- A  $x=1$  or  $x=4$
- B  $1<x<4$
- C  $x<1$  or  $x>4$
- D All real  $x$

**2.3** The minimum of  $(x+2)^2+7$  is \_\_\_\_\_. [1]

**2.4** Dividing an inequality by -2 leaves its direction unchanged. [1]

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**3 Exam-style questions**

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Use the data stated in each original task.

**3.1** Solve  $x^3 - 9x = 0$  and  $(x + 2)(x - 5) \geq 0$ . Explain why dividing the cubic by  $x$  at the start loses a solution. [3]

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**3.2** Write  $2x^2 - 12x + 23$  in vertex form and state its minimum. Find the coefficient of  $x$  in  $(2x - 3)(x + 4)$ . [3]

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**3.3** A printer charges 25 yuan to set up a job and 0.40 yuan per page. Write the total charge for  $n$  pages and solve the 65-yuan budget inequality for whole pages. Compare this with the area expression for a rectangle with sides  $x$  and  $x + 3$  metres. [3]

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## Solutions

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Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

**2.1 A** – Either factor may be zero. B loses the allowed zero root; C loses the other factor's root; D changes the sign when solving  $x-5=0$ .

Correct response linked to the given case.

**2.2 B** – The factors have opposite signs strictly between the roots. A gives zeros excluded by the strict sign; C gives positive products; D includes intervals of both signs.

Correct response linked to the given case.

**2.3 7** – The squared term is nonnegative and can equal zero.

Correct response linked to the given case.

**2.4 False** – A negative multiplier reverses order.

Correct response linked to the given case.

**3.1** Factor  $x(x-3)(x+3)=0$ , giving  $-3, 0, 3$ . Dividing by  $x$  improperly removes the allowed zero root. For the inequality, roots are  $-2, 5$ ; the product is nonnegative outside the roots, including endpoints, so  $x \leq -2$  or  $x \geq 5$ .

Sound conclusion or proposed method. Relevant supporting steps or textual evidence  
. Accurate conditions, checks or qualification.

**3.2** Completing the square gives  $2(x-3)^2+5$ , with minimum 5 at  $x=3$ . Expansion of the product is  $2x^2+5x-12$ , so the coefficient is 5.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.

**3.3**  $C = f + pn = 25 \text{ yuan} + (0.40 \text{ yuan/page})(n \text{ pages})$ . Budget  $25 + 0.40n \leq 65$  gives  $0 \leq n \leq 100$ , with integer  $n$ . The charge is affine linear in  $n$ . The rectangle has  $A = x(x+3) = x^2 + 3x$  square metres for numerical lengths in metres and  $x > 0$ ; multiplication of two variable lengths creates a quadratic, rather than a fixed fee plus a rate .

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.