8 Vectors and matrices as organised quantitative models

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS displacement vector • dot product • matrix • rate

Objective

- Add and scale vectors component by component
- Multiply a matrix and vector with matching dimensions
- Interpret entries and units in a quantitative model

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A two-dimensional vector records an ordered displacement or another paired quantity. Add corresponding components and multiply both by a scalar. The vector from A to B is B-A, not A-B. Its magnitude is $\sqrt{(u^2+v^2)}$ in a Euclidean coordinate plane; direction and magnitude are different pieces of information.

A matrix organises entries into rows and columns. Add only matrices of the same shape, entry by entry. For multiplication, the number of columns in the first factor must equal the number of rows in the second. A 2-by-3 matrix times a 3-by-1 vector produces a 2-by-1 result: each output is one row's dot product with the vector.

Order matters: matrix multiplication is generally not commutative. If A times B is defined, B times A may be undefined or produce a different result. For a row containing item quantities and a column containing unit prices in the same item order, the dot product gives the total cost. A mismatched item order produces a numerically neat but meaningless answer.

Label each component and unit before calculating. A displacement vector uses distance units, while a matrix entry may be a count, cost or rate . After multiplication, interpret the output's units. ACT problems may supply the operation definition; follow that definition rather than assuming an unfamiliar symbol always means ordinary multiplication.

A matrix uses labelled rows and columns. A quantity row $(2, 3)$ and price column $(4, 5)^T$ give cost $2(4) + 3(5) = 23$. The inner dimensions must agree before a matrix product exists. A displacement vector from A to B is $B - A$.

Quantity rows × price column (4, 5)

Row	Item A	Item B	Total cost
List 1	2	1	13
List 2	1	3	19

Quantity rows times the price column give each total; here the prices are 4 and 5

Worked example. Displacements (3,4) and (-1,2) sum to (2,6); the first has magnitude 5. Two shopping rows (2,1) and (1,3), with prices (4,5), give costs $2 \cdot 4+1 \cdot 5=13$ and $1 \cdot 4+3 \cdot 5=19$. The matrix [[2,1],[1,3]] times column [4,5] is column [13,19]. Entries represent counts times price per item.

2 Practice

2.1 (2,-3)+(4,1) is: [1]

- **A** (6,-4)
- **B** (8,-3)
- **C** (-2,-4)
- **D** (6,-2)

2.2 A 2×3 matrix times a 3×2 matrix produces: [1]

- **A** 2×2
- **B** 3×3
- **C** 2×3
- **D** 6×6

2.3 Magnitude of (6,8) is _____. [1]

2.4 AB always equals BA for matrices. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Points A and B in metres are $(1, -2)$ and $(7, 6)$. Find the displacement A to B and its magnitude. Then find the endpoint after adding displacement $(-3, 2)$ metres to B. [3]

3.2 A shop's two orders have quantity rows $(3, 2)$ and $(1, 4)$ for pens and notebooks. Prices are 2 and 7 yuan respectively. Give the matrix product and both order costs. [3]

3.3 A is 2 by 3 and B is 3 by 4. State the size of AB and whether BA is defined. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 D – Add matching components. A subtracts 1 from −3; B multiplies first components; C subtracts vectors. Addition requires 2+4 and −3+1.

Correct response linked to the given case.

2.2 A – Inner dimensions match; outer dimensions give the result shape. B uses the inner size for the output; C copies the first factor's shape; D multiplies dimensions. Only outer dimensions survive.

Correct response linked to the given case.

2.3 10 – √(36+64)=10.

Correct response linked to the given case.

2.4 False – Order generally changes the result or whether the product is defined.

Correct response linked to the given case.

3.1 $\overrightarrow{AB} = B - A = (6, 8)\,\text{m}$. Magnitude $|\overrightarrow{AB}| = \sqrt{6^2 + 8^2}\,\text{m} = 10\,\text{m}$. New endpoint $B + (-3, 2) = (4, 8)$ metres. Direction must not be reversed.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 $Q = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, $p = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ yuan per item. $Qp = \begin{pmatrix} 3(2) + 2(7) \\ 1(2) + 4(7) \end{pmatrix} = \begin{pmatrix} 20 \\ 30 \end{pmatrix}$ yuan. The item order must match the price order.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 AB has matching inner dimension 3 and outer dimensions 2 by 4. BA would require 4=2, so it is undefined. Reversing a product is not harmless.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.