

# 7 Real numbers, rational powers and complex arithmetic

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 16 marks

KEY WORDS complex conjugate • conjugate

## Objective

- Distinguish rational and irrational numbers without assuming their sums retain type
- Apply exponent and radical rules on the permitted domain
- Calculate with complex numbers using  $i^2=-1$  and conjugates

Original public practice; marks below are classroom self-check points, not official criteria.

## 1 Key knowledge and worked examples

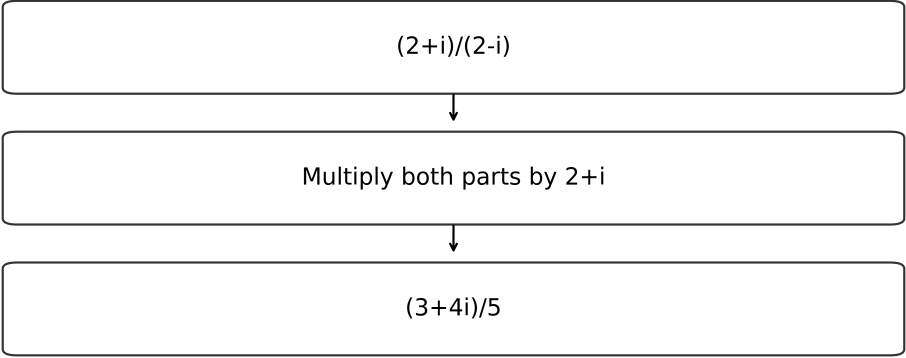
A rational number is a ratio of integers with nonzero denominator; a terminating or repeating decimal is rational. An irrational real number is not such a ratio. Closure of rational numbers under addition and multiplication does not imply closure of irrational numbers. Exact roots may simplify:  $\sqrt{18}=3\sqrt{2}$  remains irrational, whereas  $\sqrt{16}=4$  is rational.

For a positive base,  $a^{\frac{m}{n}}$  combines an  $n$ th root and an integer power. A negative exponent forms a reciprocal and requires a nonzero base. Distinguish  $\sqrt{(x^2)}=|x|$  from  $x$ . When an even root is involved, respect the real domain and the principal nonnegative root. Distribute powers across products, not across sums:  $(a+b)^2$  includes the cross term.

Complex arithmetic uses  $i^2=-1$ . Add real parts and imaginary parts separately, and multiply by distribution before reducing  $i^2$ . To divide by  $a+bi$ , multiply numerator and denominator by  $a-bi$ ; the resulting denominator is  $a^2+b^2$  when the original is nonzero. Complex roots permit solutions to equations such as  $x^2=-9$  that have no real roots.

Enhanced ACT still includes selected advanced topics, and no formula sheet is supplied. Build familiarity with the defining rules rather than guessing from a calculator display. Check whether the question asks for real or complex solutions, an exact radical or a decimal approximation. The local checks here are formative, not an official scored form.

A complex conjugate \_\_\_\_\_ changes the imaginary sign.  $(a+bi)(a-bi) = a^2+b^2$  for real  $a, b$ . Thus  $(2+i)/(2-i) = (2+i)^2/5 = (3+4i)/5$ . The denominator is nonzero; distinguish an exact value from its decimal approximation.



Worked steps:  $(2+i)/(2-i) \rightarrow$  Multiply both parts by  $2+i \rightarrow (3+4i)/5$

**Worked example.**  $16^{\frac{3}{4}}=2^3=8$ .  $(2+3i)(1-2i)=2-4i+3i-6i^2=8-i$ . For  $(1+i)/(1-i)$ , multiplying by  $1+i$  gives  $(1+i)^2/2=2i/2=i$ . The solutions of  $x^2+9=0$  are  $\pm 3i$ ; neither is real.

## 2 Practice

2.1 What is  $(3+i)(3-i)$ ? [1]

- A 9-i
- B 6
- C 10
- D 8

2.2  $81^{\frac{1}{2}}$  equals: [1]

- A 40.5
- B -9
- C 3
- D 9

2.3 The real part of  $(4+2i)+(1-5i)$  is \_\_\_\_\_. [1]

**2.4** The sum of two irrational numbers is always irrational. [1]

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**3 Exam-style questions**

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Use the data stated in each original task.

**3.1** Calculate  $(3 - 2i)/(1 + i)$  in form  $a + bi$ . [3]

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**3.2** Evaluate  $32^{2/5}$  and  $\sqrt{(-7)^2}$ . Solve  $z^2 + 16 = 0$  over the complex numbers. [3]

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**3.3** Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational. [3]

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**3.4** Classify  $\sqrt{50}$  and  $\sqrt{50}\sqrt{2}$  as rational or irrational. Evaluate  $32^{2/5}$  and  $2^{-3}$ . Use  $\sqrt{2}$  and  $-\sqrt{2}$  to test whether a sum of irrational numbers must be irrational. [3]

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## Solutions

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Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

**2.1 C** – Conjugate product is  $3^2+1^2=10$ . A fails to cancel the imaginary cross terms; B subtracts rather than multiplying; D treats  $i^2$  as  $+1$  instead of  $-1$ .

Correct response linked to the given case.

**2.2 D** – The principal square root is 9. A halves the base instead of taking its root; B chooses the negative root rather than principal value; C takes a fourth root.

Correct response linked to the given case.

**2.3 5** – Real parts add to  $4+1$ ; imaginary part is  $-3i$ .

Correct response linked to the given case.

**2.4 False** – A number plus its negative gives rational zero.

Correct response linked to the given case.

**3.1** Multiply both parts by  $1 - i$ . Numerator is  $(3 - 2i)(1 - i) = 3 - 5i + 2i^2 = 1 - 5i$ . Denominator is 2. The answer is  $1/2 - (5/2)i$ . Multiplying back by  $1 + i$  returns  $3 - 2i$ .

Sound conclusion or proposed method. Relevant supporting steps or textual evidence  
. Accurate conditions, checks or qualification.

**3.2** The fifth root of 32 is 2, so  $32^{2/5} = 2^2 = 4$ . The principal square root is  $|-7| = 7$ . For the equation,  $z^2 = -16$  gives  $z = 4i, -4i$ , both nonreal.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.

**3.3**  $\sqrt{2}\sqrt{2} = 2$  is rational.  $\sqrt{2}\sqrt{3} = \sqrt{6}$  is irrational. Neither closure nor nonclosure for all products follows from the label irrational alone.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.

**3.4**  $\sqrt{50} = 5\sqrt{2}$  is irrational, while  $\sqrt{50}\sqrt{2} = \sqrt{100} = 10$  is rational.  $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$  and  $2^{-3} = 1/2^3 = 1/8$ . The two irrational numbers sum to zero, which is rational; irrational numbers are not closed under addition. These even-root products use nonnegative real radicands.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.