

7 Real numbers, rational powers and complex arithmetic

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS complex conjugate • conjugate

Objective

- Distinguish rational and irrational numbers without assuming their sums retain type
- Apply exponent and radical rules on the permitted domain
- Calculate with complex numbers using $i^2 = -1$ and conjugates

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

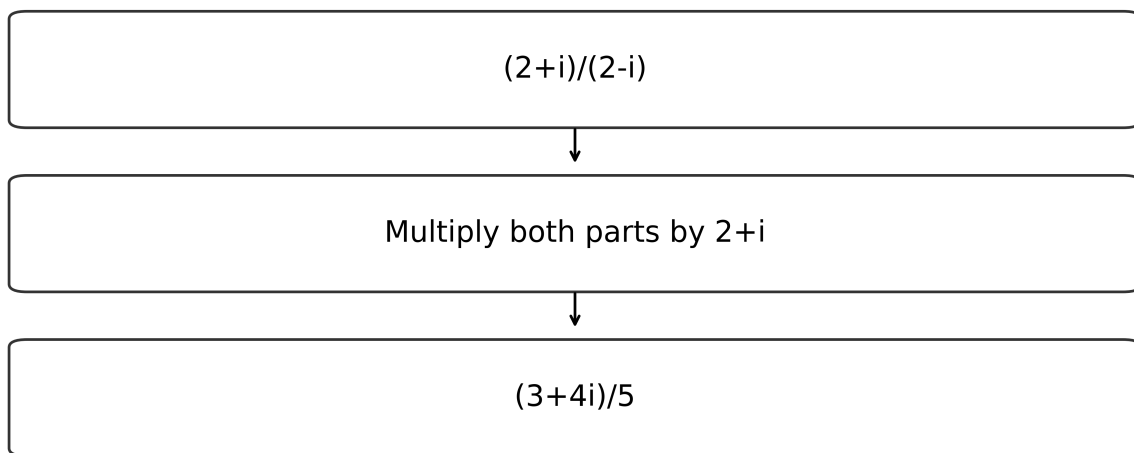
A rational number is a ratio of integers with nonzero denominator; a terminating or repeating decimal is rational. An irrational real number is not such a ratio. Closure of rational numbers under addition and multiplication does not imply closure of irrational numbers. Exact roots may simplify: $\sqrt[3]{18} = 3\sqrt[3]{2}$ remains irrational, whereas $\sqrt[3]{16} = 4\sqrt[3]{2}$ is rational.

For a positive base, $a^{m/n}$ combines an n th root and an integer power. A negative exponent forms a reciprocal and requires a nonzero base. Distinguish $\sqrt{x^2} = |x|$ from x . When an even root is involved, respect the real domain and the principal nonnegative root. Distribute powers across products, not across sums: $(a+b)^2$ includes the cross term.

Complex arithmetic uses $i^2 = -1$. Add real parts and imaginary parts separately, and multiply by distribution before reducing i^2 . To divide by $a+bi$, multiply numerator and denominator by $a-bi$; the resulting denominator is a^2+b^2 when the original is nonzero. Complex roots permit solutions to equations such as $x^2 = -9$ that have no real roots.

Enhanced ACT still includes selected advanced topics, and no formula sheet is supplied. Build familiarity with the defining rules rather than guessing from a calculator display. Check whether the question asks for real or complex solutions, an exact radical or a decimal approximation. The local checks here are formative, not an official scored form.

A complex conjugate changes the imaginary sign. $(a+bi)(a-bi) = a^2+b^2$ for real a, b . Thus $(2+i)/(2-i) = (2+i)^2/5 = (3+4i)/5$. The denominator is nonzero; distinguish an exact value from its decimal approximation.



Worked steps: $(2+i)/(2-i) \rightarrow$ Multiply both parts by $2+i \rightarrow (3+4i)/5$

Worked example. $16^{3/4}=2^3=8$. $(2+3i)(1-2i)=2-4i+3i-6i^2=8-i$. For $(1+i)/(1-i)$, multiplying by $1+i$ gives $(1+i)^2/2=2i/2=i$. The solutions of $x^2+9=0$ are $\pm 3i$; neither is real.

2 Practice

2.1 What is $(3+i)(3-i)$? [1]

- A 9-i
- B 6
- C 10
- D 8

2.2 $81^{1/2}$ equals: [1]

- A 40.5
- B -9
- C 3
- D 9

2.3 The real part of $(4+2i)+(1-5i)$ is _____. [1]

2.4 The sum of two irrational numbers is always irrational. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Calculate $(3 - 2i)/(1 + i)$ in form $a + bi$. [3]

3.2 Evaluate $32^{2/5}$ and $\sqrt{(-7)^2}$. Solve $z^2 + 16 = 0$ over the complex numbers. [3]

3.3 Give one example showing that the product of two irrational numbers can be rational, and one where it is irrational. [3]

3.4 Classify $\sqrt{50}$ and $\sqrt{50}\sqrt{2}$ as rational or irrational. Evaluate $32^{2/5}$ and 2^{-3} . Use $\sqrt{2}$ and $-\sqrt{2}$ to test whether a sum of irrational numbers must be irrational. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 C – Conjugate product is $3^2 + 1^2 = 10$. A fails to cancel the imaginary cross terms; B subtracts rather than multiplying; D treats i^2 as $+1$ instead of -1 .

Correct response linked to the given case.

2.2 D – The principal square root is 9. A halves the base instead of taking its root; B chooses the negative root rather than principal value; C takes a fourth root.

Correct response linked to the given case.

2.3 5 – Real parts add to $4+1$; imaginary part is $-3i$.

Correct response linked to the given case.

2.4 False – A number plus its negative gives rational zero.

Correct response linked to the given case.

3.1 Multiply both parts by $1 - i$. Numerator is $(3 - 2i)(1 - i) = 3 - 5i + 2i^2 = 1 - 5i$. Denominator is 2. The answer is $1/2 - (5/2)i$. Multiplying back by $1 + i$ returns $3 - 2i$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 The fifth root of 32 is 2, so $32^{2/5} = 2^2 = 4$. The principal square root is $|-7| = 7$. For the equation, $z^2 = -16$ gives $z = 4i, -4i$, both nonreal.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 $\sqrt{2}\sqrt{2} = 2$ is rational. $\sqrt{2}\sqrt{3} = \sqrt{6}$ is irrational. Neither closure nor nonclosure for all products follows from the label irrational alone.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.4 $\sqrt{50} = 5\sqrt{2}$ is irrational, while $\sqrt{50}\sqrt{2} = \sqrt{100} = 10$ is rational. $32^{2/5} = (\sqrt[5]{32})^2 = 2^2 = 4$ and $2^{-3} = 1/2^3 = 1/8$. The two irrational numbers sum to zero, which is rational; irrational numbers are not closed under addition. These even-root products use nonnegative real radicands.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.