18 Essential skills in a multi-step practical model

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS evidence • rate • residual

Objective

- Build a model connecting quantities, rates and geometric measures
- Interpret parameters and test the model's assumptions
- Evaluate predictions and adjust a parameter using new evidence

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

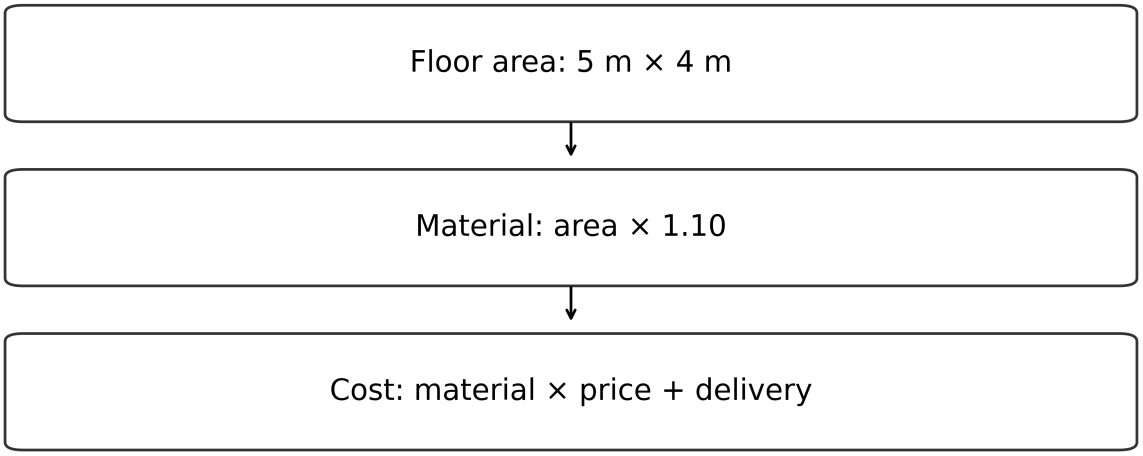
Define variables with units and translate each part of the situation. A fixed fee plus a rate gives C=b+md; direct proportion C=md applies only when the intercept is zero. Ratios compare quantities in a chosen order, and successive percentage changes multiply their changing bases. A model should preserve those relationships before any arithmetic begins.

Convert units by multiplying factors that cancel. For scale models, length factors square for area and cube for volume when every dimension is scaled. Combining a geometric formula with a unit price can model material cost; for example, paint area times cost per square metre gives currency, while using wall volume would have the wrong physical meaning.

Interpret and test the model. Ask whether a constant rate is plausible across the domain, whether a fixed charge is already included and whether an estimated area excludes doors or overlap. Compare a prediction with observed data and identify which assumption or parameter may need revision. A model's neat algebra does not establish that it fits reality.

Improve a model by adjusting the parameter supported by evidence. If every predicted bill is exactly two units low, revising the fixed fee may fit better than changing the per-distance slope. A trend in errors with distance points toward the rate instead. Keep modelling as an overlapping activity across content areas, not a separate extra block of official questions.

A rate compares quantities with their units. $v = d/t = (90\,\text{km})/(1.5\,\text{h}) = 60\,\text{km/h}$. Convert using cancelling units: $v = (60\,\text{km/h})(1000\,\text{m/km})/(3600\,\text{s/h}) = 50/3\,\text{m/s}$. Do not convert only the numerator of a compound rate.



Worked steps: Floor area: 5 m × 4 m → Material: area × 1.10 → Cost: material × price + delivery

Worked example. Original delivery model C=8+3d, d in kilometres, predicts 23 units for 5 km. Bills are consistently 2 units higher at several distances, supporting a revised intercept 10 while keeping slope 3. A wall 4 m by 2.5 m has area 10 m^2; subtracting a 2 m^2 doorway gives 8 m^2 to paint. At 6 units/m^2, cost is 48. A 1:50 plan of a 4 m wall shows 8 cm.

2 Practice

2.1 C=5+2d. For d=7, C is: [1]

- **A** 12
- **B** 19
- **C** 14
- **D** 35

2.2 Every prediction is 3 units too low across several distances. First adjustment to inspect: [1]

- **A** Squaring every distance
- **B** Deleting the data
- **C** The fixed intercept
- **D** Changing distance units without conversion

2.3 A 3 m length on a 1:100 plan is _____ cm. [1]

2.4 Modelling is counted only in a separate block of enhanced ACT math items. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 A room is 5 m by 4 m. Allow extra flooring equal to 10% of the room area for wastage, costs 35 yuan per square metre, and delivery adds 60 yuan. Build a cost model and find the total. A 1:50 plan shows the room; give its dimensions in centimetres. [3]

3.2 A delivery model $C = 6 + 4d$ yuan is consistently 5 yuan below observed bills at distances 2,4,6 km. Propose a first revision. Explain what pattern instead would suggest a slope error. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 B – Fixed fee 5 plus $2 \cdot 7$=19. A adds distance to fee; C omits the fixed fee; D multiplies fee by distance. None evaluates 5+2×7.

Correct response linked to the given case.

2.2 C – A constant offset suggests the fixed contribution is low. A changes shape despite a constant offset; B discards useful evidence; D changes units inconsistently. An intercept check targets the observed fixed error.

Correct response linked to the given case.

2.3 3 – 3 m=300 cm, divided by 100.

Correct response linked to the given case.

2.4 False – It overlaps other reporting categories.

Correct response linked to the given case.

3.1 Area $A = LW = (5\,\mathrm{m})(4\,\mathrm{m}) = 20\,\mathrm{m}^2$. Required material $Q = 1.10A = 22\,\mathrm{m}^2$. Cost $C = pQ + d = (35\,\mathrm{yuan/m}^2)(22\,\mathrm{m}^2) + 60\,\mathrm{yuan} = 830\,\mathrm{yuan}$. Plan dimensions are $500/50 = 10$ cm and $400/50 = 8$ cm. Wastage is on area, not on both side lengths.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.2 A constant residual suggests raising the intercept by 5 to $C = 11 + 4d$. A residual systematically growing with distance would suggest checking the per-kilometre rate. Neither pattern alone proves the reason; check fee definitions and other conditions.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.