

17 Sampling inference, expected value and counting

Name: _____ Class: _____ Date: _____

Total: 22 marks

KEY WORDS conditional probability • margin of error
 • expected value • inference • evidence

Objective

- Interpret a population estimate and simulation evidence without claiming proof
- Calculate conditional, compound and expected-value quantities
- Choose permutations or combinations according to whether order matters

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A random sample supports inference to its source population; a margin of error gives a sampling-uncertainty interval around an estimate. Random assignment instead supports a causal treatment comparison. In a simulation, a result rarely produced under a claim can be evidence against that claim; failure to find an unusual result does not prove the claim true.

Conditional probability restricts the denominator to the conditioning group. For either A or B, subtract their overlap after adding separate probabilities. Independence permits multiplication of unchanged probabilities; without-replacement draws require updated counts. Expected value is the sum of each possible outcome times its probability, not necessarily an outcome achieved in one trial.

Use the multiplication principle for successive choices. For n distinct objects, choosing r in order gives $n!/(n-r)!$; choosing an unranked set gives $n!/r!(n-r)!$. Dividing by $r!$ removes the multiple orderings of the same set. These formulas assume distinct objects and no replacement; different conditions need a corresponding count.

Translate the event to a count before calculating its probability. If a favourable event can occur in several disjoint ways, add those ways; if paths overlap, avoid double-counting. A numerical answer must be between zero and one for a probability, while an expected cost or score has the units of the outcomes.

A margin of error is added and subtracted in percentage points. Estimate 63% with margin 4 points gives 59%–67%. It is an interval from a procedure, not a guarantee that each person's response is near 63%. Random sampling supports population inference; random assignment addresses treatment comparisons.

Choose 2 of 4: six unordered pairs; AB and BA count once

AB	AC	AD
BC	BD	CD

Choosing 2 of 4 gives six pairs, because AB and BA are the same pair

Worked example. From five students, captain/deputy pairs number $5 \cdot 4 = 20$; unranked pairs number $5 \cdot 4/2 = 10$. A game awards 4 points with probability $1/4$ and 0 otherwise, so expected award is 1 point per play. A sample estimate $52\% \pm 3$ percentage points gives 49%–55%; it is not a guarantee about every sample or individual.

2 Practice

2.1 Choose two unranked representatives from four people. Number of choices: [1]

- A 6
- B 12
- C 8
- D 16

2.2 A random award is 10 with probability 0.2 and 0 otherwise. Expected award: [1]

- A 0.2
- B 2
- C 10
- D 5

2.3 An estimate $64\% \pm 4$ points has lower endpoint _____. [1]

2.4 Failing to reject a claim establishes that it is certainly true. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Six students choose a chairperson and a different secretary. How many assignments? How many unranked two-person committees? Explain the difference. [3]

3.2 A game pays 12 tokens with probability 0.1, 3 with probability 0.4, and zero otherwise. Entry costs 2 tokens. Find expected payout and expected net gain. Does a player receive that amount every time? [3]

3.3 Two balls are drawn without replacement from a bag with 3 red and 2 blue balls. Find the probability both are red and of exactly one red. [3]

3.4 A random sample of 500 registered library members reports 62% support for a change, with stated margin 4 percentage points. State the interval and population. Does it establish support among every town resident? [3]

3.5 Forty volunteers are randomly assigned to old or new instructions, with the same task and conditions. The new group finishes faster. What causal and population claims are reasonable? [3]

3.6 A poll of 10000 website volunteers has a narrow reported margin. Explain why size alone cannot repair selection bias, and propose a better recruitment method. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 **A** – $4 \cdot 3/2=6$; order does not matter. **B** counts ordered roles; **C** doubles a count without correct pairing; **D** allows repeated independent selections and order. Neither is an unranked pair of distinct people.

Correct response linked to the given case.

2.2 **B** – $10 \cdot 0.2+0 \cdot 0.8=2$. **A** is probability without award size; **C** is the possible prize, not its average; **D** assumes two equally likely outcomes despite probability 0.2.

Correct response linked to the given case.

2.3 60 – Subtract four percentage points.

Correct response linked to the given case.

2.4 False – It means the evidence did not justify rejection under the procedure.

Correct response linked to the given case.

3.1 Ordered roles give $6(5) = 30$. Unranked pairs count each pair twice in that product, so there are $6(5)/2 = 15$. Swapping people changes roles but not an unranked committee.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.2 Remaining probability is 0.5. $E(P) = 12(0.1)+3(0.4)+0(0.5) = 2.4$ tokens. Expected net is $E(G) = E(P) - 2 = 0.4$ token per play. Actual net outcomes are 10,1,-2; none equals 0.4, which is a long-run mean under the model.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 $P(RR) = (3/5)(2/4) = 3/10$. Exactly one red can occur as RB or BR: $P = (3/5)(2/4) + (2/5)(3/4) = 3/5$. The second denominator is 4; the draws are dependent.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.4 Interval is $62\% - 4\% = 58\%$ to $62\% + 4\% = 66\%$, where the subtraction denotes points. The population is registered members in the sampling frame. Nonmembers were not sampled, so the estimate does not establish all residents' support.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.5 Random assignment supports a causal comparison of the instruction versions for these participants under the tested conditions, subject to variation. Volunteer selection limits population generalisation. It does not establish that every future user will finish faster.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.6 People choosing to respond can differ systematically from those who do not. A margin for sampling variability does not remove that bias. Select randomly from the defined target population and follow up nonresponse; still report limitations.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.