

15 Conics, circle relationships and geometric measurement

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS hyperbola • semiaxes • parabola • ellipse • secant

Objective

- Identify and interpret standard circle, ellipse, parabola and hyperbola forms
- Use circle angle, tangent and secant relationships under their conditions
- Calculate area, surface area or volume with the correct dimension and units

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

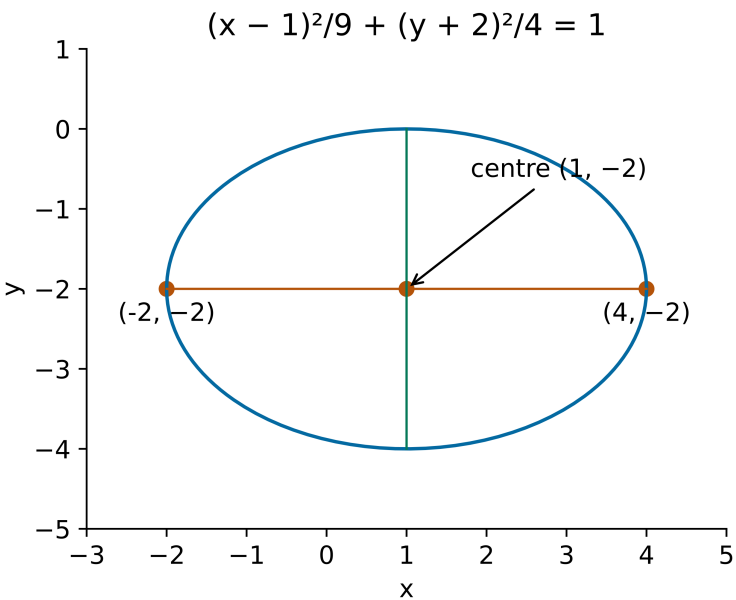
A circle has form $(x-h)^2+(y-k)^2=r^2$. An ellipse uses two positive squared terms divided by squared semiaxes; a hyperbola has a difference of such terms. A parabola has one squared coordinate in its standard aligned form. In $y^2=4px$, the vertex is the origin and the focus is $(p,0)$; read the sign and axis before interpreting direction.

For $(x-1)^2/9+(y+2)^2/4=1$, centre is $(1,-2)$, horizontal semiaxis 3 and vertical semiaxis 2. Complete the square when the centre is hidden in an expanded circle. An equation's coefficients and signs distinguish conic types; a rough sketch does not supply missing algebraic information.

A tangent is perpendicular to the radius at contact. An inscribed angle is half the central angle subtending the same arc. For two secants from the same exterior point, outside segment times whole secant is equal for both; a tangent length squared equals that product. Whole length includes the outside portion, so do not multiply outside and inside alone.

Separate boundary measures from space measures. Circle circumference is $2\pi r$ and area πr^2 ; cylinder volume is $\pi r^2 h$; cone volume is $\pi r^2 h/3$; sphere volume is $4\pi r^3/3$. Surface area sums exposed faces. If all corresponding lengths scale by k , area scales by k^2 and volume by k^3 ; changing only one dimension needs the original formula.

An ellipse $(x-h)^2/a^2+(y-k)^2/b^2=1$ has centre (h,k) . Semiaxes are square roots of the positive denominators. A parabola $y^2=4px$ has focus $(p,0)$ and directrix $x=-p$. Distinguish these equations from a hyperbola with a difference of squares.



The denominators 9 and 4 give semi-axes 3 across and 2 up and down

Worked example. The ellipse $(x-1)^2/9+(y+2)^2/4=1$ has horizontal endpoints $(-2,-2)$ and $(4,-2)$. $y^2=8x$ has $p=2$ and focus $(2,0)$. Secants with outside/whole lengths $3/12$ and $4/9$ both have product 36, so a tangent from the same point has length 6. A cone radius 3, height 8 has volume 24π .

2 Practice

2.1 For $x^2/16+y^2/9=1$, horizontal semiaxis: [1]

- A 8
- B 3
- C 4
- D 16

2.2 A cone of radius 2 and height 9 has volume: [1]

- A 36π
- B 18π
- C 6π
- D 12π

2.3 Radius of $(x-2)^2+(y+1)^2=25$ is _____. [1]

2.4 A tangent-radius angle at contact is 90° . [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 For $(x + 2)^2/25 + (y - 1)^2/9 = 1$, state centre, horizontal and vertical semiaxes, and four axis endpoints. For $y^2 = -12x$, state vertex and focus. [3]

3.2 From one exterior point a tangent has length 6 cm. A secant has outside segment 4 cm. Find its whole and inside lengths. [3]

3.3 A cylindrical container has radius 3 cm and height 8 cm. Find its volume and total closed surface area. What happens to each if every length doubles? [3]

3.4 Classify $x^2/16 - y^2/9 = 1$ and $x^2 + y^2 = 16$. Find the circle circumference and area. For two secants from the same exterior point, one has outside/whole lengths $3/12$ cm and the other outside length 4 cm: find its whole and inside lengths. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 C – The denominator is the semiaxis squared. A doubles the semiaxis into an axis length; B reads the vertical denominator's root; D uses the squared semiaxis.

Correct response linked to the given case.

2.2 D – $\pi \cdot 2^2 \cdot 9/3 = 12\pi$. A gives cylinder volume without the factor one third; B uses an incorrect radius contribution; C understates the cone volume by a factor of two.

Correct response linked to the given case.

2.3 5 – Radius squared is 25.

Correct response linked to the given case.

2.4 True – They are perpendicular at that point.

Correct response linked to the given case.

3.1 Centre is $(-2, 1)$, semiaxes 5 and 3, endpoints $(-7, 1), (3, 1), (-2, -2), (-2, 4)$. In the parabola, $4p = -12$, so $p = -3$; vertex $(0, 0)$, focus $(-3, 0)$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
. Accurate conditions, checks or qualification.

3.2 Use tangent-secant relation $t^2 = ew$. Rearrange $w = t^2/e = (6 \text{ cm})^2/(4 \text{ cm}) = 9 \text{ cm}$. Inside length is $w - e = 9 \text{ cm} - 4 \text{ cm} = 5 \text{ cm}$. The product uses whole length, not just inside.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 $V = \pi r^2 h = \pi(3 \text{ cm})^2(8 \text{ cm}) = 72\pi \text{ cm}^3$. $A = 2\pi r^2 + 2\pi r h = 2\pi(3 \text{ cm})^2 + 2\pi(3 \text{ cm})(8 \text{ cm}) = 66\pi \text{ cm}^2$. Doubling all lengths multiplies volume by 8 and area by 4.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.4 The difference of squared terms defines a hyperbola centred at the origin. The sum with equal coefficients defines a circle of radius 4. Its circumference is $C = 2\pi r = 8\pi$ and area $A = \pi r^2 = 16\pi$ in coordinate length and square-length units respectively. Secant products match: $e_1 w_1 = e_2 w_2$. Thus $w_2 = (3 \text{ cm})(12 \text{ cm})/(4 \text{ cm}) = 9 \text{ cm}$; inside length is $9 - 4 = 5 \text{ cm}$. Outside times inside would apply the wrong relation.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.