14 Plane transformations, coordinate reasoning and proof

Name: _____ Class: _____ Date: _____ **Total: 19 marks**

KEY WORDS perpendicular bisector • rigid motions • translation • similarity • congruence • evidence • argument • transformation

Objective

- Transform coordinates while distinguishing rigid motions from dilation
- Use distance, midpoint and slope as geometric evidence
- Use valid congruence and angle relationships rather than a diagram's appearance

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

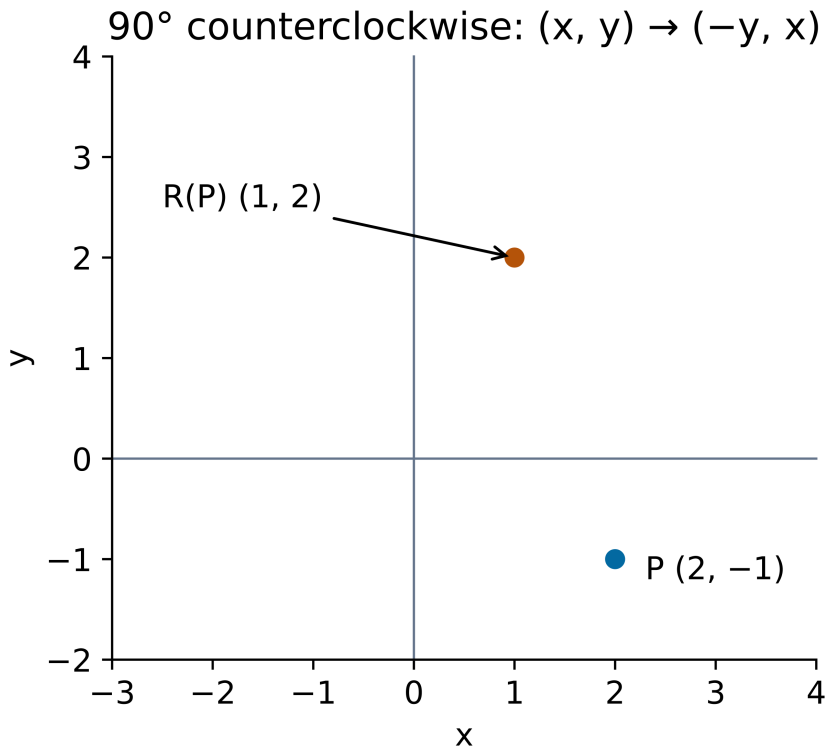
A translation adds a displacement to each point; reflection across the x-axis sends (x,y) to (x,-y). A 90° counterclockwise rotation around the origin sends (x,y) to (-y,x). These rigid motions preserve lengths and angles. Dilation by factor k about the origin scales coordinates and lengths by k when k is positive, preserving shape but generally changing size.

Distance between two points follows the Pythagorean theorem: square the coordinate differences, add and take the root. The midpoint averages each coordinate. Slope compares vertical and horizontal change; perpendicular nonvertical slopes have product -1. Vertical and horizontal lines require separate treatment rather than division by zero.

A geometric argument starts from stated or marked conditions. Vertical angles are equal; parallel lines give corresponding and alternate-angle relationships. Triangle angles sum to 180°. SSS, SAS and ASA/AAS can establish congruence with proper correspondence, while two equal angles establish similarity but not equal size.

Construction reasoning uses a compass to transfer lengths and draw equal-radius arcs. An intersection of equal-distance arcs can locate a perpendicular bisector ; it is not a guessed midpoint from a sketch. Coordinate evidence can support a proof, but one specially chosen picture does not establish a general statement for every triangle.

Rigid motions preserve lengths and angles. A 90-degree anticlockwise rotation sends (x, y) to $(-y, x)$ about the origin. Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Equal angles establish similarity , but congruence also fixes size.



A 90° counterclockwise rotation sends (x, y) to (−y, x)

Worked example. Point (2,-1) translated by (3,4) becomes (5,3); rotated 90° counterclockwise about the origin it becomes (1,2). Between (1,2) and (7,10), distance is $\sqrt{(6^2+8^2)}=10$ and midpoint (4,6). Two triangles with matching side lengths 3,4,5 are congruent by SSS; triangles with equal angles may instead differ by scale.

2 Practice

2.1 Rotate (3,2) 90° counterclockwise about the origin. [1]

- **A** (3,-2)
- **B** (-2,3)
- **C** (2,-3)
- **D** (-3,2)

2.2 Midpoint of (0,2) and (6,8): [1]

- **A** (3,3)
- **B** (0,8)
- **C** (3,5)
- **D** (6,10)

2.3 A line of slope 2 has a perpendicular nonvertical slope _____. [1]

2.4 Two equal triangle angles prove congruence. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Rotate A = $(2, -3)$ by 90° anticlockwise about the origin, then translate by $(4, 1)$. Reverse the order and compare endpoints. [3]

3.2 A $= (-1, 2)$ and B $= (5, 10)$ are endpoints of a segment. Find its length, midpoint and perpendicular-bisector equation. [3]

3.3 Two triangles have angles 40°,60°,80°. One has longest side 9 cm; the other 15 cm. State what is proved and the ratio of their areas. [3]

3.4 Describe a compass-and-straightedge construction of the perpendicular bisector of segment AB. Give a reason valid for every constructed point, and distinguish it from measuring one diagram. Then explain what SSS proves for triangles with corresponding side lengths 3,4,5 cm. [3]

3.5 Two parallel lines are cut by a transversal. One corresponding angle is $68°$. Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 B – Apply (-y,x). A reflects in the horizontal axis; C rotates clockwise; D reflects in the vertical axis. None is the stated anticlockwise quarter-turn.

Correct response linked to the given case.

2.2 C – Average corresponding coordinates. A averages the y difference rather than y coordinates; B combines endpoints without averaging; D adds coordinates without halving.

Correct response linked to the given case.

2.3 -1/2 – The slopes multiply to -1.

Correct response linked to the given case.

2.4 False – They establish similarity; lengths may differ.

Correct response linked to the given case.

3.1 Rotate first to $(3, 2)$, then translate to $(7, 3)$. Translate first to $(6, -2)$, then rotate to $(2, 6)$. The endpoints differ, so transformation order matters.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.2 $d = \sqrt{(5+1)^2 + (10-2)^2} = 10$. Midpoint $M = ((-1+5)/2, (2+10)/2) = (2, 6)$. Segment slope is $8/6 = 4/3$, so perpendicular slope is $-3/4$. The bisector is $y - 6 = -(3/4)(x - 2)$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 The triangles are similar by equal angles, not congruent because corresponding longest sides differ. Length ratio is $15/9 = 5/3$ and larger-to-smaller area ratio is $25/9$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.4 Use the same compass radius, greater than half AB, to draw arcs centred at A and B meeting at P and Q on opposite sides. Draw PQ. PA=PB and QA=QB by equal radii. Thus both P and Q lie on the locus equidistant from A and B; PQ is the perpendicular bisector. Equivalently use congruent triangles and the common chord to establish perpendicularity and bisection. A ruler reading on one picture does not prove the general locus. SSS proves the two stated triangles congruent, fixing size as well as shape; Pythagoras also shows these particular triangles are right-angled.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.5 Parallelism gives the matching corresponding angle 68°. The adjacent pair forms a straight angle, so the other is 180°−68°=112°. Equal corresponding angles require the

stated parallel lines; the straight-angle sum uses adjacency on one line. A similar-looking sketch without parallelism would not justify the first equality.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.