

13 Trigonometric graphs, identities and general triangles

Name: _____ Class: _____ Date: _____ Total: 16 marks

KEY WORDS included angle • cosine rule • amplitude

Objective

- Interpret amplitude, period and vertical shift in a sinusoidal rule
- Use unit-circle signs and basic identities
- Choose a sine, cosine or area relation for a general triangle

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

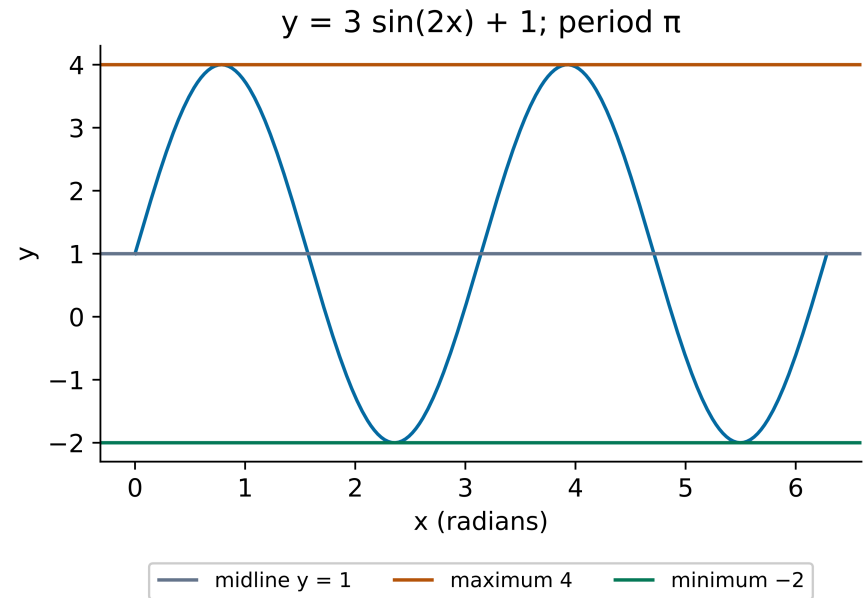
For a right triangle, sin is opposite/hypotenuse, cos adjacent/hypotenuse and tan opposite/adjacent relative to a chosen angle. Similar triangles explain why these ratios depend on angle rather than size. On the unit circle, coordinates are $(\cos \theta, \sin \theta)$, extending the ratios to other quadrants with appropriate signs.

$\sin^2 \theta + \cos^2 \theta = 1$ and $\tan \theta = \sin \theta / \cos \theta$ when $\cos \theta \neq 0$. For $y = A \sin(Bx) + D$ with radian x , amplitude is $|A|$, period $2\pi/|B|$ and midline D . A negative A reflects the wave; D shifts it vertically. Check whether the input is in radians or degrees before using a graph interval.

For a general triangle, the cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$ uses the included angle C between a and b . The sine rule compares each side with the sine of its opposite angle. Area is $ab \sin C / 2$ when two sides and their included angle are known. Choose the relationship matching the supplied information.

Do not infer a right angle from the sketch. A sine-based equation may allow more than one angle in a permitted interval; retain the quadrant information and any triangle sum condition. Keep amplitude distinct from total peak-to-trough distance, which is twice the amplitude.

Amplitude is half the peak-to-trough distance. In $y = A \sin(Bx) + D$, it is $|A|$; radian period is $2\pi/|B|$. For a non-right triangle, match known sides and included angle before selecting the cosine rule. $c^2 = a^2 + b^2 - 2ab \cos C$.



Amplitude 3 about the midline $y = 1$; the factor 2 makes the period π

Worked example. $y = 3 \sin(2x) + 1$ has amplitude 3, period π and values between -2 and 4 . A triangle with sides 5 and 7 around a 60° angle has third side $\sqrt{(25+49-70 \cdot 1/2)} = \sqrt{39}$ and area $35\sqrt{3}/4$. At 150° , sine is $1/2$ and cosine $-\sqrt{3}/2$.

2 Practice

2.1 $y = 2 \sin(3x) + 4$ has maximum: [1]

- A 6
- B 2
- C 4
- D 8

2.2 For radian x , $\sin(2x)$ has period: [1]

- A $1/2$
- B π
- C 2π
- D 4π

2.3 $\sin^2\theta + \cos^2\theta =$ _____. [1]

2.4 The Pythagorean theorem directly applies to every triangle. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 For radian input $y = -4\sin(2x) + 3$, give amplitude, period, range and value at $x = \pi/4$. [3]

3.2 A triangle has sides 6 cm and 10 cm enclosing 60° . Find its third side and area without assuming it is right-angled. [3]

3.3 Find every angle in $0^\circ \leq \theta < 360^\circ$ with $\sin \theta = 1/2$. For each, give cosine and tangent. [3]

3.4 A right triangle has legs 9 and 12 cm. Find the hypotenuse and sine/cosine of the angle opposite 9 cm. A different triangle has angles 30° and 45° and side 8 cm opposite 30° . Find the side opposite 45° , explaining why the sine rule fits. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – Midline 4 plus amplitude 2 is 6. B gives amplitude; C gives midline; D double-counts amplitude. Maximum is midline plus one amplitude.

Correct response linked to the given case.

2.2 B – $2\pi/2=\pi$. A gives reciprocal frequency without the full-turn factor; C ignores the input multiplier; D multiplies the old period instead of dividing.

Correct response linked to the given case.

2.3 1 – This follows from the unit-circle equation .

Correct response linked to the given case.

2.4 False – It requires a right angle.

Correct response linked to the given case.

3.1 Amplitude is 4, period $2\pi/2 = \pi$, range $-1 \leq y \leq 7$. At $x = \pi/4$, $y = -4 \sin(\pi/2) + 3 = -1$. The negative coefficient reflects the wave; it does not make amplitude negative.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 $c^2 = a^2 + b^2 - 2ab \cos C = (6 \text{ cm})^2 + (10 \text{ cm})^2 - 2(6 \text{ cm})(10 \text{ cm}) \cos 60^\circ = 76 \text{ cm}^2$, so $c = 2\sqrt{19} \text{ cm}$. $A = ab \sin C/2 = (6 \text{ cm})(10 \text{ cm}) \sin 60^\circ/2 = 15\sqrt{3} \text{ cm}^2$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 Angles are 30 and 150 degrees. Cosines are $\sqrt{3}/2$ and $-\sqrt{3}/2$; tangents are $1/\sqrt{3}$ and $-1/\sqrt{3}$. Sine is positive in quadrants I and II, so one acute answer would be incomplete.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.4 $c = \sqrt{(9 \text{ cm})^2 + (12 \text{ cm})^2} = 15 \text{ cm}$. Thus $\sin \theta = 9/15 = 3/5$ and $\cos \theta = 12/15 = 4/5$; their squares sum to 1. In the second triangle a known opposite side-angle pair permits $b/\sin 45^\circ = (8 \text{ cm})/\sin 30^\circ$. Hence $b = (8 \text{ cm})(\sqrt{2}/2)/(1/2) = 8\sqrt{2} \text{ cm}$. This triangle is not assumed right-angled; its third angle is 105° .

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.