12 Sequences, finite sums, exponentials and logarithms

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS arithmetic sequence • geometric sequence • logarithm • argument

Objective

- Distinguish arithmetic difference from geometric ratio
- Find a term or finite sum while keeping indexing consistent
- Convert logarithmic equations to exponential form on the valid domain

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

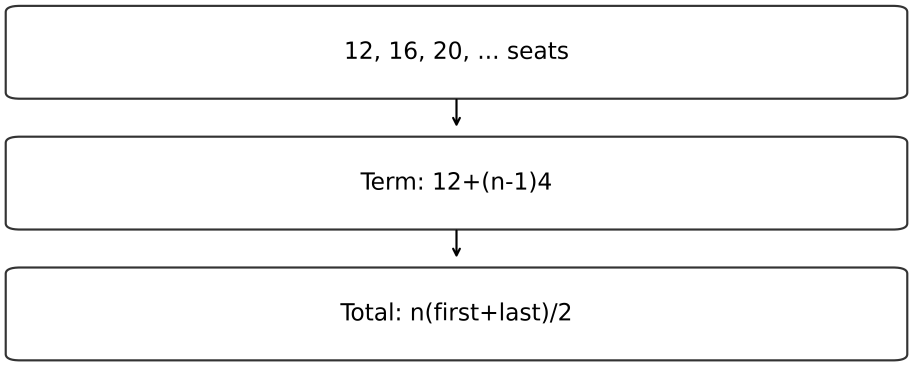
An arithmetic sequence has constant difference d: $a_n=a_1+(n-1)d$. Its finite sum is $n(a_1+a_n)/2$. A geometric sequence has constant ratio r: $a_n=a_1r^{(n-1)}$. For $r\neq1$, a finite sum is $a_1(1-r^n)/(1-r)$; when r=1 it is simply $n\,a_1$. Match the requested term or total.

An exponential model A b^t represents multiplication by b per stated time unit. A percentage growth r uses factor 1+r; repeated changes multiply factors rather than adding percentages. If doubling takes several time units, divide t by that interval in the exponent.

For base b>0 with b≠1, log_b(a)=c means b^c=a and requires a>0. Solve a logarithmic equation by translating it to an exponential relationship, then check the argument . Logarithms of products add under valid positive arguments; logarithms do not distribute over addition.

Use exact powers when available. ACT may test a simple logarithmic relationship without requiring numerical logarithm tables. A calculated approximation must still respect the question's representation and domain. Keep a sequence's index, an exponential model's time and a logarithm's argument conceptually separate.

Arithmetic sequence : $a_n = a_1 + (n-1)d$. Its sum is $S_n = n(a_1 + a_n)/2$. Geometric sequence : $a_n = a_1 r^{n-1}$ and $S_n = a_1(1-r^n)/(1-r)$ for $r \neq 1$. A logarithm reverses exponentiation: $\log_b a = c$ means $b^c = a$, with $a > 0$, $b > 0$, $b \neq 1$.



Worked steps: 12, 16, 20, … seats → Term: 12+(n-1)4 → Total: n(first+last)/2

Worked example. Arithmetic 4,7,10,… has a_5=16 and sum of first five=5(4+16)/2=50. Geometric 3,6,12,24 has four-term sum 45. $\log_2$(x-1)=4 gives x-1=16, so x=17, which satisfies x>1. A population 50 · 2^(t/3) reaches 200 after six time units.

2 Practice

2.1 Arithmetic sequence starts at 5 with difference 4. Fourth term: [1]

- **A** 21
- **B** 9
- **C** 20
- **D** 17

2.2 $\log_3$ 81 equals: [1]

- **A** 4
- **B** 3
- **C** 27
- **D** 9

2.3 The first three terms 2,6,18 have sum _____. [1]

2.4 log(a+b)=log a+log b for all positive a,b. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Seats in successive rows number 12,16,20,… . Find the eighth row and the total in the first eight rows. State the model assumption. [3]

3.2 A geometric sequence starts 5,15,45,… . Find its fifth term and sum of the first five terms. Explain what changes if the ratio is 1. [3]

3.3 Solve $\log_2(x-3)=5$ and explain why $\log_2(x+3)$ cannot generally be split into $\log_2 x+\log_2 3$. [3]

3.4 A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t, find $N(4)$, and distinguish factor from percentage growth. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 D – $5+3 \cdot 4=17$. A counts four increments instead of three; B gives the second term; C multiplies initial value by term number instead of adding the common difference.

Correct response linked to the given case.

2.2 A – $3^4=81$. B gives the logarithm's base; C divides 81 by 3 once; D is not the exponent producing 81 from base 3.

Correct response linked to the given case.

2.3 26 – Add the three terms; 18 alone is the last term.

Correct response linked to the given case.

2.4 False – The product rule concerns ab, not a+b.

Correct response linked to the given case.

3.1 Constant difference is 4 seats per row. $a_8 = a_1 + 7d = 12 + 7(4) = 40$ seats. $S_8 = 8(a_1+a_8)/2 = 8(12+40)/2 = 208$ seats. This assumes the same four-seat increment through all eight rows.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 $a_5 = a_1r^4 = 5(3)^4 = 405$. $S_5 = a_1(1-r^5)/(1-r) = 5(1-243)/(1-3) = 605$. When $r = 1$, the fraction has zero denominator and the correct sum is na_1, here 25 for five identical terms.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 The domain requires $x > 3$. Exponential form gives $x - 3 = 32$, so $x = 35$ and the argument is positive. The sum of logs corresponds to the product $3x$, not $x+3$; at $x = 1$, the proposed equality would equate 2 with $\log_2 3$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.4 $N(t) = 120(1.5)^{t/2}$ cells. $N(4) = 120(1.5)^2 = 270$ cells. The growth is 50% per two hours, not 150%; the four-hour factor is 2.25. The hourly factor would be $\sqrt{1.5}$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.