

# 11 Functions, composition, transformations and inverses

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 13 marks

KEY WORDS transformation • composition • one-to-one

## Objective

- Evaluate and convert between function rules, tables and graphs
- Track input order and domain through composition
- Find an inverse after choosing a one-to-one domain

Original public practice; marks below are classroom self-check points, not official criteria.

## 1 Key knowledge and worked examples

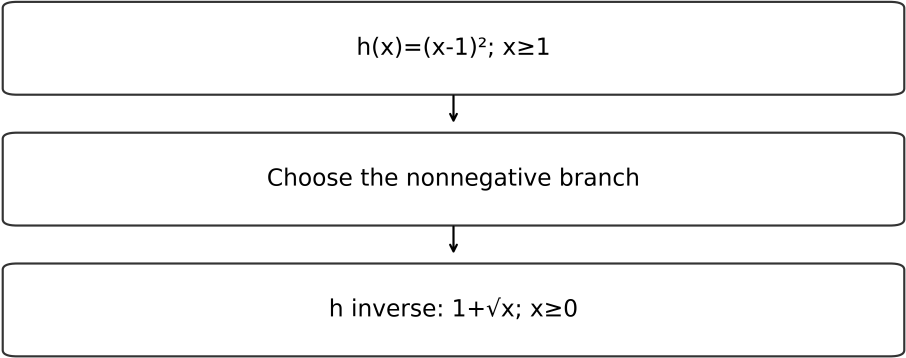
A function gives one output for each allowed input. A graph passing a vertical line at two different output points is not a function of  $x$ . Domain lists allowed inputs; range lists attained outputs. Convert a table to a rule only when the relationship is supported, and interpret each variable's role.

In  $f(g(x))$ , the inner function  $g$  acts first and its output must be in  $f$ 's domain. For  $f(u)=1/(u-1)$  and  $g(x)=x^2+1$ , composition gives  $1/x^2$  and excludes  $x=0$ . Domain checking belongs to the original functions as well as the simplified result.

An output change  $f(x)+k$  shifts vertically by  $k$ ;  $f(x-h)$  shifts right by  $h$ . An outside multiplier changes output scale, while an inside multiplier changes how quickly the input reaches old points. Identify a known point to verify the transformation instead of guessing direction from a sign.

An inverse exchanges input and output and requires a one-to-one relationship on the chosen domain. Solve  $y=f(x)$  for  $x$ , then exchange names. A quadratic must usually be restricted to one side of its vertex. The inverse function  $f^{-1}$  is not the reciprocal  $1/f$ . Check composition in both directions on the allowed domain.

Composition applies the inside rule first. If  $f(u) = 1/(u-2)$  and  $g(x) = x^2+2$ ,  $f(g(x)) = 1/x^2$ , with  $x \neq 0$ . An inverse reverses input and output on a one-to-one domain. Restrict  $h(x) = (x-1)^2$  to  $x \geq 1$  before taking  $h^{-1}(x) = 1 + \sqrt{x}$ .



Worked steps:  $h(x)=(x-1)^2; x \geq 1 \rightarrow$  Choose the nonnegative branch  $\rightarrow$   $h$  inverse:  $1+\sqrt{x}; x \geq 0$

**Worked example.**  $f(x)=2x+3$  and  $g(x)=x^2$  give  $f(g(x))=2x^2+3$ , but  $g(f(x))=(2x+3)^2$ . For  $h(x)=(x-2)^2$  restricted to  $x \geq 2$ , the inverse is  $2+\sqrt{x}$  with domain  $x \geq 0$ . A vertical shift of  $h$  by 4 has vertex  $(2,4)$ ; the unrestricted quadratic would not have a single-valued inverse.

## 2 Practice

2.1  $f(x)=3x+1$ ,  $g(x)=x-2$ .  $f(g(5))$  equals: [1]

- A 7
- B 16
- C 10
- D 14

2.2 The inverse of  $f(x)=4x-8$  is: [1]

- A  $1/(4x-8)$
- B  $4x+8$
- C  $(x-8)/4$
- D  $(x+8)/4$

2.3 The vertex of  $(x-5)^2+2$  has x-coordinate \_\_\_\_\_. [1]

**2.4** An unrestricted quadratic has a single-valued inverse on all real outputs. [1]

**3 Exam-style questions**

Use the data stated in each original task.

**3.1** Let  $f(u) = 1/(u - 3)$  and  $g(x) = x^2 + 3$ . Find  $f(g(x))$  with domain. Find  $g(f(4))$  and explain why these compositions differ. [3]

**3.2** Restrict  $h(x) = (x + 2)^2$  to  $x \leq -2$ . Find its inverse, including domain and range. Explain why the positive-root branch is wrong. [3]

**3.3** The graph of  $f(x) = x^2$  is changed to  $g(x) = 3f(x + 4) - 2$ . Find its vertex and range, checking one mapped point. [3]

## Solutions

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Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

**2.1 C** –  $g(5)=3$ , then  $f(3)=10$ . A has no correct inside-then-outside substitution; B stops at  $f(5)$ ; D computes  $g(f(5))$ , reversing the order.

Correct response linked to the given case.

**2.2 D** – Solve  $y=4x-8$  for  $x=(y+8)/4$ . A is a reciprocal rather than an inverse; B does not undo multiplication by 4; C subtracts 8 where solving requires adding it.

Correct response linked to the given case.

**2.3 5** – The horizontal shift is right by 5.

Correct response linked to the given case.

**2.4 False** – Two different inputs can share an output; a branch restriction is needed.

Correct response linked to the given case.

**3.1**  $f(g(x)) = 1/(x^2 + 3 - 3) = 1/x^2$ , excluding  $x = 0$ . For the other order,  $f(4) = 1$  and  $g(1) = 4$ . Composition order changes both operation and possibly domain;  $f(g(4)) = 1/16$  is different.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence  
. Accurate conditions, checks or qualification.

**3.2** Solve  $y = (x + 2)^2$  with  $x + 2 \leq 0$  to get  $x = -2 - \sqrt{y}$ . Thus  $h^{-1}(x) = -2 - \sqrt{x}$ , domain  $x \geq 0$ , range  $y \leq -2$ . The plus branch returns values outside the chosen original domain.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.

**3.3**  $g(x) = 3(x + 4)^2 - 2$  has vertex  $(-4, -2)$  and range  $y \geq -2$ . Old point  $(1, 1)$  maps to  $(-3, 1)$  because  $3(1) - 2 = 1$ ; substitution confirms  $g(-3) = 1$ .

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.  
Accurate conditions, checks or qualification.