

11 Functions, composition, transformations and inverses

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS transformation • composition • one-to-one

Objective

- Evaluate and convert between function rules, tables and graphs
- Track input order and domain through composition
- Find an inverse after choosing a one-to-one domain

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

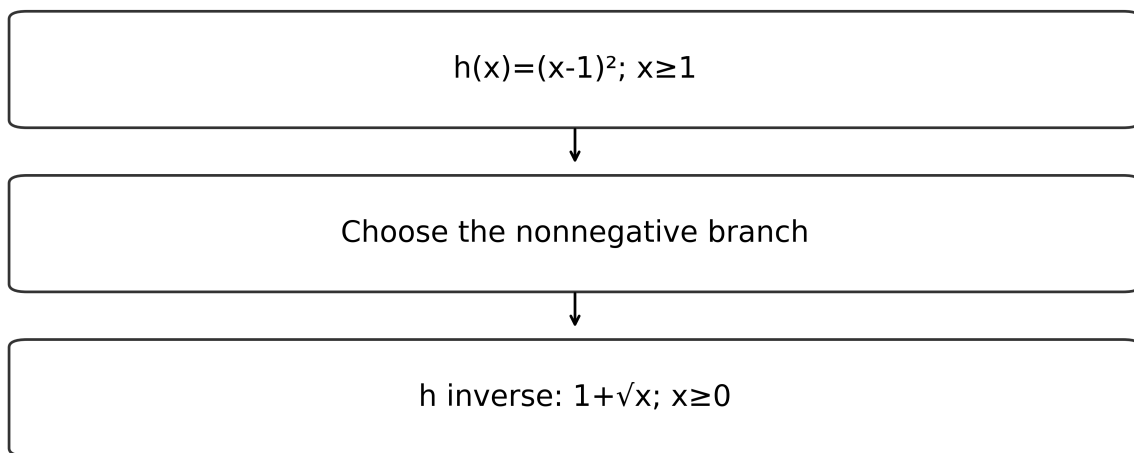
A function gives one output for each allowed input. A graph passing a vertical line at two different output points is not a function of x . Domain lists allowed inputs; range lists attained outputs. Convert a table to a rule only when the relationship is supported, and interpret each variable's role.

In $f(g(x))$, the inner function g acts first and its output must be in f 's domain. For $f(u)=1/(u-1)$ and $g(x)=x^2+1$, composition gives $1/x^2$ and excludes $x=0$. Domain checking belongs to the original functions as well as the simplified result.

An output change $f(x)+k$ shifts vertically by k ; $f(x-h)$ shifts right by h . An outside multiplier changes output scale, while an inside multiplier changes how quickly the input reaches old points. Identify a known point to verify the transformation instead of guessing direction from a sign.

An inverse exchanges input and output and requires a one-to-one relationship on the chosen domain. Solve $y=f(x)$ for x , then exchange names. A quadratic must usually be restricted to one side of its vertex. The inverse function f^{-1} is not the reciprocal $1/f$. Check composition in both directions on the allowed domain.

Composition applies the inside rule first. If $f(u) = 1/(u-2)$ and $g(x) = x^2+2$, $f(g(x)) = 1/x^2$, with $x \neq 0$. An inverse reverses input and output on a one-to-one domain. Restrict $h(x) = (x-1)^2$ to $x \geq 1$ before taking $h^{-1}(x) = 1 + \sqrt{x}$.



Worked steps: $h(x)=(x-1)^2; x \geq 1 \rightarrow$ Choose the nonnegative branch $\rightarrow$ h inverse: $1+\sqrt{x}; x \geq 0$

Worked example. $f(x)=2x+3$ and $g(x)=x^2$ give $f(g(x))=2x^2+3$, but $g(f(x))=(2x+3)^2$. For $h(x)=(x-2)^2$ restricted to $x \geq 2$, the inverse is $2+\sqrt{x}$ with domain $x \geq 0$. A vertical shift of h by 4 has vertex $(2,4)$; the unrestricted quadratic would not have a single-valued inverse.

2 Practice

2.1 $f(x)=3x+1$, $g(x)=x-2$. $f(g(5))$ equals: [1]

- A 7
- B 16
- C 10
- D 14

2.2 The inverse of $f(x)=4x-8$ is: [1]

- A $1/(4x-8)$
- B $4x+8$
- C $(x-8)/4$
- D $(x+8)/4$

2.3 The vertex of $(x-5)^2+2$ has x-coordinate _____. [1]

2.4 An unrestricted quadratic has a single-valued inverse on all real outputs. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Let $f(u) = 1/(u - 3)$ and $g(x) = x^2 + 3$. Find $f(g(x))$ with domain. Find $g(f(4))$ and explain why these compositions differ. [3]

3.2 Restrict $h(x) = (x + 2)^2$ to $x \leq -2$. Find its inverse, including domain and range. Explain why the positive-root branch is wrong. [3]

3.3 The graph of $f(x) = x^2$ is changed to $g(x) = 3f(x + 4) - 2$. Find its vertex and range, checking one mapped point. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 C – $g(5)=3$, then $f(3)=10$. A has no correct inside-then-outside substitution; B stops at $f(5)$; D computes $g(f(5))$, reversing the order.

Correct response linked to the given case.

2.2 D – Solve $y=4x-8$ for $x=(y+8)/4$. A is a reciprocal rather than an inverse; B does not undo multiplication by 4; C subtracts 8 where solving requires adding it.

Correct response linked to the given case.

2.3 5 – The horizontal shift is right by 5.

Correct response linked to the given case.

2.4 False – Two different inputs can share an output; a branch restriction is needed.

Correct response linked to the given case.

3.1 $f(g(x)) = 1/(x^2 + 3 - 3) = 1/x^2$, excluding $x = 0$. For the other order, $f(4) = 1$ and $g(1) = 4$. Composition order changes both operation and possibly domain; $f(g(4)) = 1/16$ is different.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Solve $y = (x + 2)^2$ with $x + 2 \leq 0$ to get $x = -2 - \sqrt{y}$. Thus $h^{-1}(x) = -2 - \sqrt{x}$, domain $x \geq 0$, range $y \leq -2$. The plus branch returns values outside the chosen original domain.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 $g(x) = 3(x + 4)^2 - 2$ has vertex $(-4, -2)$ and range $y \geq -2$. Old point $(1, 1)$ maps to $(-3, 1)$ because $3(1) - 2 = 1$; substitution confirms $g(-3) = 1$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.