

10 Rational equations, radical branches and systems

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS candidate root • parabola

Objective

- Record denominator and radical restrictions before solving
- Check transformed-equation candidates in the original
- Find and interpret simultaneous solutions or allowed regions

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

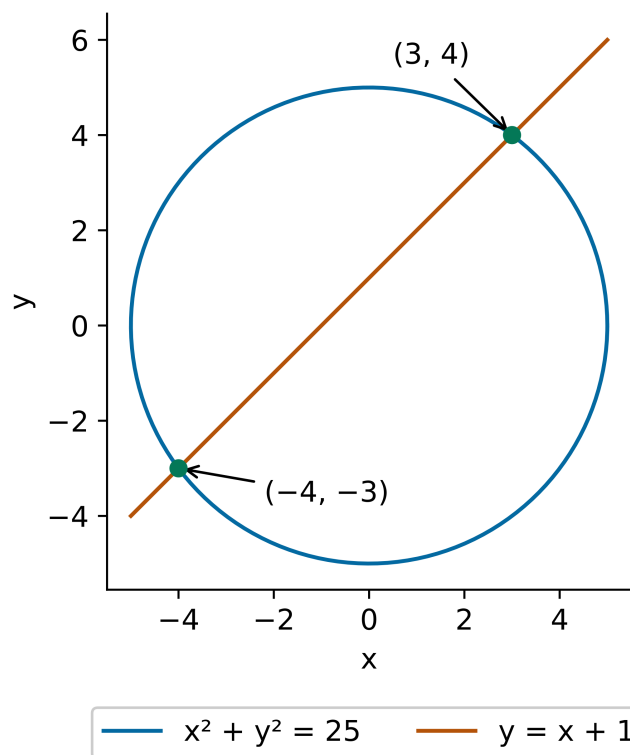
For rational expressions, identify denominator zeros before cancelling factors or multiplying an equation by a denominator. Simplification may remove a visible factor while its original restriction remains. Cancel whole factors, not selected terms in an addition.

Isolate a square root and respect its nonnegative value before squaring. Squaring preserves solutions but can add candidates, so substitute each candidate into the original. The same caution applies to multiplying by an expression that might be zero. A value that satisfies a transformed polynomial but fails the source is extraneous.

A system requires the same ordered pair to satisfy every equation. Substitute a line into a circle or parabola, solve the resulting quadratic and recover the second coordinate. Linear systems may have one intersection, no intersection or coincident lines. Do not confuse two algebraic roots with two separate systems.

For systems of inequalities, each condition supplies an allowed region and the solution is their intersection. A point on a strict boundary is excluded even if it satisfies the other inequality. Verify a proposed point by substituting into each original inequality, including its equality sign.

A candidate root must satisfy the original equation. In $\sqrt{x+6} = x$, the right side must be nonnegative. Squaring gives $x^2 - x - 6 = 0$, so candidates are 3, -2. Only 3 survives the original check: $\sqrt{9} = 3$.



Substituting the line into the circle gives both crossing points

Worked example. $\sqrt{x+6}=x$ implies $x \geq 0$. Squaring gives $x^2 - x - 6 = 0$ with candidates 3 and -2; only 3 works. For $x^2 + y^2 = 25$ and $y = x + 1$, substitution gives $x^2 + x - 12 = 0$, hence (3,4) and (-4,-3). $(x^2 - 4)/(x - 2)$ simplifies to $x + 2$ for $x \neq 2$, preserving the restriction.

2 Practice

2.1 Which solves $\sqrt{x+2}=x$?

[1]

- A Neither
- B 2
- C -1
- D Both 2 and -1

2.2 $(x^2 - 1)/(x - 1)$ has the original restriction:

[1]

- A $x=1$
- B No restriction
- C $x \neq 1$
- D $x \neq -1$

2.3 For $x+y=9$ and $x-y=3$, $x=$ _____. [1]

2.4 A point satisfying one inequality must satisfy their whole system. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Solve simultaneously $x^2 + y^2 = 25$ and $y = x - 1$. Check each ordered pair in both equations. [3]

3.2 Solve $\sqrt{3x+4} = x$ and describe all points satisfying $y \geq x + 1$ and $y < 4 - x$. [3]

3.3 Simplify $F(x) = (x^2 - 4x)/(x^2 - 16)$. State every original restriction and explain why F is not identical to $x/(x + 4)$ on that latter formula's whole domain. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 B – Only nonnegative 2 satisfies the original equation. A discards the valid root 2; C has a negative right side and fails the principal root; D retains that extraneous negative candidate.

Correct response linked to the given case.

2.2 C – The denominator is zero at 1. A requires the prohibited input; B forgets the original denominator; D excludes a numerator zero rather than the denominator zero.

Correct response linked to the given case.

2.3 6 – Adding gives $2x=12$; $y=3$.

Correct response linked to the given case.

2.4 False – Every condition must hold simultaneously.

Correct response linked to the given case.

3.1 Substitute to get $x^2 + (x - 1)^2 = 25$, or $x^2 - x - 12 = 0$. Roots are 4, -3 , giving $(4, 3)$ and $(-3, -4)$. Both give squared sum 25 and satisfy $y = x - 1$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Squaring with $x \geq 0$ gives $(x - 4)(x + 1) = 0$; only $x = 4$ works. The region is on/above $y = x + 1$ and below $y = 4 - x$. It exists only when $x + 1 < 4 - x$, so $x < 3/2$; their meeting point is excluded.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 Factor numerator $x(x - 4)$ and denominator $(x - 4)(x + 4)$. The original domain excludes $x = 4, -4$. Cancellation gives $F(x) = x/(x + 4)$ for $x \neq 4, -4$. The latter formula alone permits 4, where it gives $1/2$, while $F(4)$ is undefined.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.