

Exercise walk-through

15

G-conics · Conics, circle relationships and geometric measurement

ACT · ACT · ACT

15 Conics, circle relationships and geometric measurement

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS hyperbola • semiaxes • parabola • ellipse • secant

Objective

- Identify and interpret standard circle, ellipse, parabola and hyperbola forms
- Use circle angle, tangent and secant relationships under their conditions
- Calculate area, surface area or volume with the correct dimension and units

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A circle has form $(x-h)^2+(y-k)^2=r^2$. An ellipse uses two positive squared terms divided by squared semiaxes; a hyperbola has a difference of such terms. A parabola has

one squared coordinate in its standard aligned form. In $y^2=4px$, the vertex is the origin and the focus is $(p,0)$; read the sign and axis before interpreting direction.

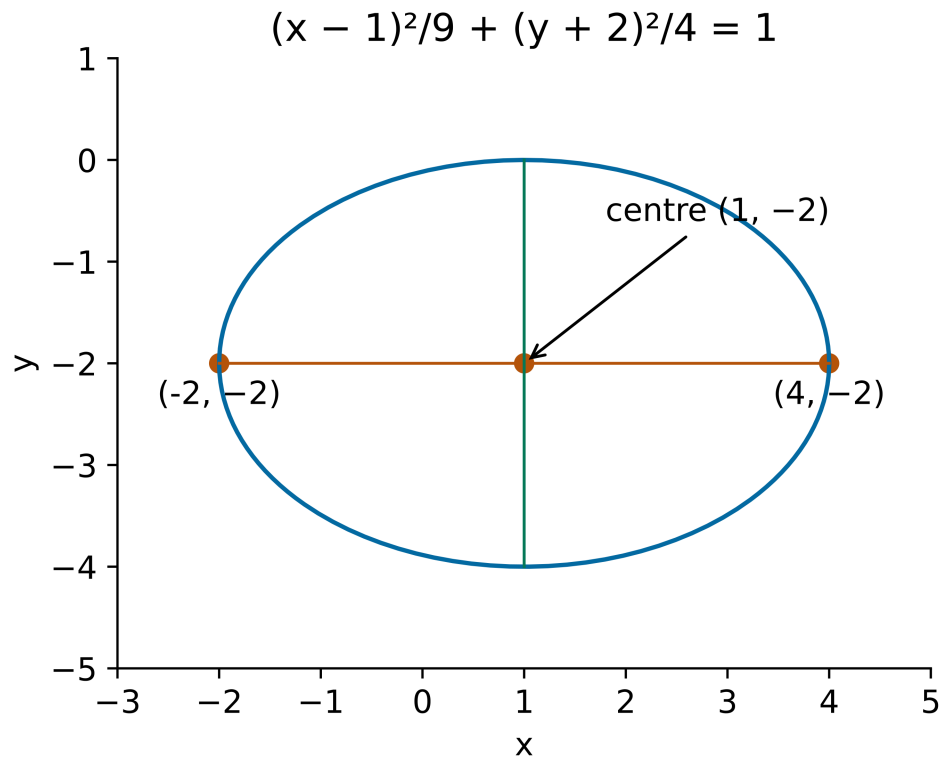
For $(x-1)^2/9+(y+2)^2/4=1$, centre is $(1,-2)$, horizontal semiaxis 3 and vertical semiaxis 2. Complete the square when the centre is hidden in an expanded circle. An equation's coefficients and signs distinguish conic types; a rough sketch does not supply missing algebraic information.

A tangent is perpendicular to the radius at contact. An inscribed angle is half the central angle subtending the same arc. For two secants from the same exterior point, outside segment times whole secant is equal for both; a tangent length squared equals that product. Whole length includes the outside portion, so do not multiply outside and inside alone.

Separate boundary measures from space measures. Circle circumference is $2\pi r$ and area πr^2 ; cylinder volume is $\pi r^2 h$; cone volume is $\pi r^2 h/3$; sphere volume is $4\pi r^3/3$. Surface

area sums exposed faces. If all corresponding lengths scale by k , area scales by k^2 and volume by k^3 ; changing only one dimension needs the original formula.

An ellipse $(x - h)^2/a^2 + (y - k)^2/b^2 = 1$ has centre (h, k) . Semiaxes are square roots of the positive denominators. A parabola $y^2 = 4px$ has focus $(p, 0)$ and directrix $x = -p$. Distinguish these equations from a hyperbola with a difference of squares.



Original worked example from existing native teaching; transfer tasks use their own data.

Worked example from the native lesson: The ellipse $(x-1)^2/9+(y+2)^2/4=1$ has horizontal endpoints $(-2,-2)$ and $(4,-2)$. $y^2=8x$ has $p=2$ and focus $(2,0)$. Secants with outside/whole lengths $3/12$ and $4/9$ both have product 36, so a tangent from the same point has length 6. A cone radius 3, height 8 has volume 24π .

Exercise 2.1

2.1 For $x^2/16 + y^2/9 = 1$, horizontal semiaxis:

[1]

- **A** 8
- **B** 3
- **C** 4
- **D** 16

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[1]

- **A** 8
- **B** 3
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- **D** 16

2.1 C – The denominator is the semiaxis squared. A doubles the semiaxis into an axis length; B reads the vertical denominator's root; D uses the squared semiaxis.

Correct response linked to the given case.

Exercises 2.2·2.3

2.2 A cone of radius 2 and height 9 has volume:

[1]

- A 36π
- B 18π
- C 6π
- D 12π

2.3 Radius of $(x-2)^2+(y+1)^2=25$ is _____.

[1]

Exercises 2.2·2.3

2.2 A cone of radius 2 and height 9 has volume:

[1]

- A 36π
- B 18π
- C 6π
- D 12π

2.2 D – $\pi \cdot 2^2 \cdot 9/3 = 12\pi$. A gives cylinder volume without the factor one third; B uses an incorrect radius contribution; C understates the cone volume by a factor of two.

Correct response linked to the given case.

2.3 Radius of $(x-2)^2 + (y+1)^2 = 25$ is _____.

[1]

2.3 5 – Radius squared is 25.

Correct response linked to the given case.

Exercises 2.4·3.1

2.4 A tangent-radius angle at contact is 90° .

[1]

3.1 For $(x + 2)^2/25 + (y - 1)^2/9 = 1$, state centre, horizontal and vertical semiaxes, and four axis endpoints. For $y^2 = -12x$, state vertex and focus.

[3]

Exercises 2.4·3.1

2.4 A tangent-radius angle at contact is 90° . [1]

2.4 True – They are perpendicular at that point.

Correct response linked to the given case.

3.1 For $(x + 2)^2/25 + (y - 1)^2/9 = 1$, state centre, horizontal and vertical semiaxes, and four axis endpoints. For $y^2 = -12x$, state vertex and focus. [3]

3.1 Centre is $(-2, 1)$, semiaxes 5 and 3, endpoints $(-7, 1)$, $(3, 1)$, $(-2, -2)$, $(-2, 4)$. In the parabola, $4p = -12$, so $p = -3$; vertex $(0, 0)$, focus $(-3, 0)$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

Exercises 3.2-3.3

3.2 From one exterior point a tangent has length 6 cm. A secant has outside segment 4 cm. Find its whole and inside lengths. [3]

3.3 A cylindrical container has radius 3 cm and height 8 cm. Find its volume and total closed surface area. What happens to each if every length doubles? [3]

Exercises 3.2·3.3

3.2 From one exterior point a tangent has length 6 cm. A secant has outside segment 4 cm. Find its whole and inside lengths. [3]

3.2 Use tangent–secant relation $t^2 = ew$. Rearrange $w = t^2/e = (6\text{ cm})^2/(4\text{ cm}) = 9\text{ cm}$. Inside length is $w - e = 9\text{ cm} - 4\text{ cm} = 5\text{ cm}$. The product uses whole length, not just inside.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 A cylindrical container has radius 3 cm and height 8 cm. Find its volume and total closed surface area. What happens to each if every length doubles? [3]

3.3 $V = \pi r^2 h = \pi(3\text{ cm})^2(8\text{ cm}) = 72\pi\text{ cm}^3$. $A = 2\pi r^2 + 2\pi r h = 2\pi(3\text{ cm})^2 + 2\pi(3\text{ cm})(8\text{ cm}) = 66\pi\text{ cm}^2$. Doubling all lengths multiplies volume by 8 and area by 4.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

Exercise 3.4

3.4 Classify $x^2/16 - y^2/9 = 1$ and $x^2 + y^2 = 16$. Find the circle circumference and area. For two secants from the same exterior point, one has outside/whole lengths 3/12 cm and the other outside length 4 cm: find its whole and inside lengths. [3]

Exercise 3.4

3.4 Classify $x^2/16 - y^2/9 = 1$ and $x^2 + y^2 = 16$. Find the circle circumference and area. For two secants from the same exterior point, one has outside/whole lengths 3/12 cm and the other outside length 4 cm: find its whole and inside lengths. [3]

3.4 The difference of squared terms defines a hyperbola centred at the origin. The sum with equal coefficients defines a circle of radius 4. Its circumference is $C = 2\pi r = 8\pi$ and area $A = \pi r^2 = 16\pi$ in coordinate length and square-length units respectively. Secant products match: $e_1 w_1 = e_2 w_2$. Thus $w_2 = (3 \text{ cm})(12 \text{ cm})/(4 \text{ cm}) = 9 \text{ cm}$; inside length is $9 - 4 = 5 \text{ cm}$. Outside times inside would apply the wrong relation.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.